AUM SAI INSTITUTE OF TECHNICAL EDUCATION
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DEPARTMENT OF ELECTRICAL ENGINEERING

CIRCUIT & NETWORK THEORY

3rd SEMESTER

LECTURE NOTE

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CHAPTER1

CircuitElementsandLaws

Voltage

Energy is required for the movement of charge from one point to another. Let W Joules of energy be required to move positive charge Q columbs from a point a to point b in a circuit. We say that a voltage exists between the two points. The voltageV between two points may be defined in terms of energy that would be required if a charge were transferred from one point to the other. Thus, there can be a voltage between two points even if no charge is actually moving from one to the other. Voltage between a and b is given by

$$V=\frac{W}{Q}\ J/C$$

$$\text{HenceElectricPotential}(V)=\frac{\text{Workedare}(W)\text{in Joules}}{\text{Charge}(Q)\text{incolumbs}}$$

Current:

An electric current is the movement of electric charges along a definite path. In caseof a conductor the moving charges are electrons.

The unit of current is the ampere. The ampere is defined as that current which when flowing in two infinitely long parallel conductors of negligible crosssection, situated 1meter apart in Vacuum, produces between the conductors a force of 2 x 10^{-7} Newton per metre length.

Power : Power is defined as the work done per unit time. If a field F newton acts for t seconds through adistance dmetres alonga straight line, work done W = Fxd N.m. or J. The power P, either generated or dissipated by the circuit element.

$$P=\frac{w}{t}=\frac{Fxd}{t}$$

Powercan also bewrittenasPower= $\frac{Work}{time}$

$= \frac{Work}{Charge} \times \frac{Charge}{Time} =$ VoltagexCurrent

P=VxIwatt.

Energy: Electric energy W is defined as the Power Consumed in a given time. Hence, if current IAflowsin an element overatimeperiod tsecond, when avoltageVvoltsisapplied across it, the energy consumed is given by

W =Pxt= VxIxtJorwatt.second.

The unit of energy W is Joule (J) or watt. second. However, in practice, the unit of energy is kilowatt. hour (Kwh)

Resistance: AccordingtoOhm's lawpotentialdifference (V)across theends of aconductor is proportional to the current (I) flowing through the conductorata constant temperature. Mathematically Ohm's law is expressed as

$V \alpha I$ or $V=R \times I$

Or $R= \frac{V}{I}$ whereRistheproportionalityconstantandisdesignatedastheconductor resistanceandhas the unitofOhm(Ω).

Conductance : Voltage is inducedin astationaryconductor when placed ina varying magnetic field. The induced voltage (e) is proportional to the time rate of change of current, di/dt producing the magnetic field.

Therefore $e \alpha \frac{di}{dt}$

Or $e=L\frac{di}{dt}$

eandiarebothfunctionoftime.TheproportionalityconstantLiscalledinductance. TheUnitofinductance isHenery(H).

Capacitance: A capacitor is a Physical device, which when polarized by an electric field by applying a suitable voltage across it, storesenergy in the form of a charge separation.

Theabilityofthecapacitortostorechargeismeasuredintermsofcapacitance. Capacitenceofacapacitorisdefinedasthechargestoredper Voltapplied.

$$C=\frac{q}{v}=\frac{\text{Coulomb}}{\text{Volt}}=\text{Farad}$$

ActiveandpassiveBranch:

A branch is said to be active when it contains one or more energy sources. A passive branch does not contain an energy source.

Branch: Abranchisanelementofthe networkhaving onlytwoterminals.

Bilateralandunilateralelement:

A bilateral element conducts equally well in either direction. Resistors and inductors are examples of bilateralelements. When the current voltage relations are different for the two directions of current flow, the element is said to be unilateral. Diode is an unilateral element.

Linear Elements: When the current and voltage relationship in an element can be simulated by a linear equation either algebraic, differential or integral type, the element is said to be linear element.

Non Linear Elements : When the current and voltage relationship in an element can not besimulated by a linear equation, the element is said to be non linear elements.

Kirchhoff'sVoltageLaw(KVL):

ThealgebraicsumofVoltages(orvoltagedrops)in any closedpathorloopisZero.

Application of KVL with series connected voltage source.

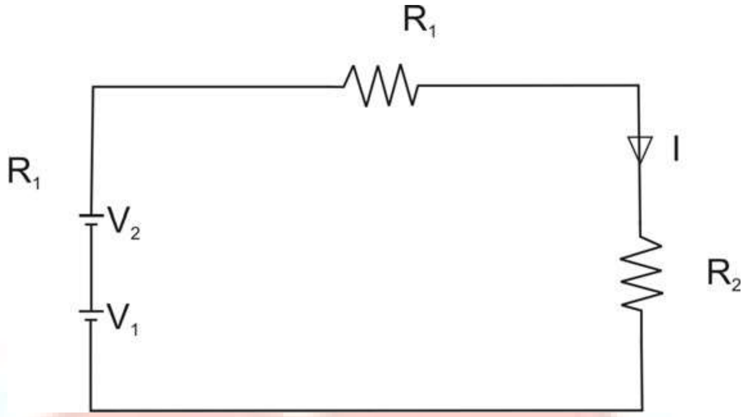


Fig.1.1

$V_1+V_2-IR_1-IR_2=0$

$=V_1+V_2=I(R_1+R_2)$

$I=\frac{V_1+V_2}{R_1+R_2}$

Application of KVL while voltage sources are connected in opposite polarity.

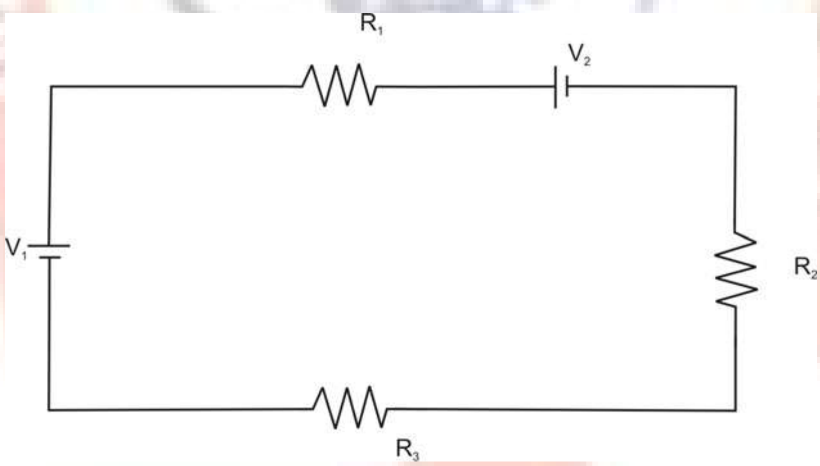


Fig.1.2

$V_1-IR_1-V_2-IR_2-IR_3=0$

- $V_1-V_2=IR_1+IR_2+IR_3$
- $V_1-V_2=I(R_1+IR_2+IR_3)$

- $$I = \frac{V_1 - V_2}{R_1 + R_2 + R_3}$$

Kirchaoff's CurrentLaw(KCL):

Thealgebraicsumofcurrentsmeetingat ajunction ormodeiszero.

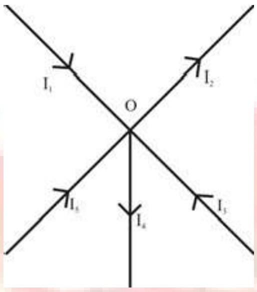


Fig.1.3

Considering five conductors, carrying currents I_1, I_2, I_3, I_4andI_5meeting at a point O. Assuming the incoming currents to be positive and outgoing currents negative.

$I_1+(-I_2)+I_3+(-I_4)+I_5=0$ I_1-

$I_2+ I_3- I_4+ I_5= 0$

$I_1+I_3+I_5=I_2+ I_4$

Thus above Law can also be stated as the sum of currents flowing towards any junction in an electric circuit is equal to the sum of the currents flowing away from that junction.

VoltageDivision(SeriesCircuit)

Considering avoltagesource(E)withresistors$R_1$and R_2inseriesacrossit.

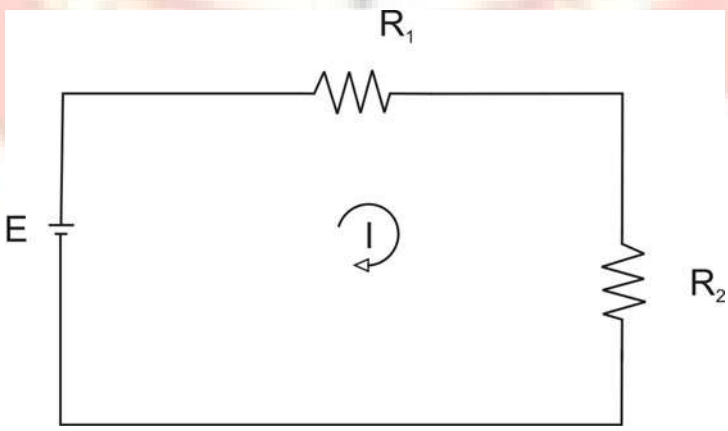


Fig.1.4

$$I = \frac{ER_1}{+R_2}$$

VoltagedropacrossR_1=I.R_1= $\frac{E.R_1}{R_1+R_2}$

Similarlyvoltagedrop acrossR_2=I.R_2= $\frac{E.R_1}{R_1+R_2}$

CurrentDivision:

A parallelcircuitactsas acurrentdivideras the currentdivides inallbranches ina parallel circuit.

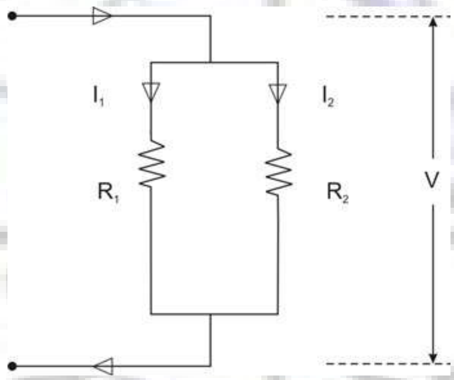


Fig.1.5

Fig.shownthecurrentIhasbeendividedintoI_1andI_2intwoparallelbrancheswithresistances $R_1$and R_2while V is the voltage drop across $R_1$and R_2.

$I_1=\frac{V}{R_1}$ and$I_2=\frac{V}{R_2}$

LetR=Totalresistance ofthe circuit.

Hence $\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}$

- ➢ $R=\frac{R_1R_2}{R_1+R_2}$

$$I=\frac{V}{R}=\frac{V}{\frac{R_1R_2}{R_1+R_2}}=\frac{V(R_1+R_2)}{R_1R_2}$$

But = $V=I_1R_1=I_2R_2$

- $I=I_1R_1\left(\frac{R_1R_2}{R_1+R_2}\right)$

- $I=\frac{I_1(R_1+R_2)}{R_2}$

Therefore $\boxed{I_1=\frac{IR_2}{R_1+R_2}}$

Similarlyitcanbederivedthat

$\boxed{I_2=\frac{IR_1}{R_1+R_2}}$

CHAPTER2

MagneticCircuits:

Introduction: Magnetic flux lines always form closed loops. The closed path followed by the flux lines is called a magnetic circuit. Thus, a magnetic circuit provides a path for magnetic flux, just as an electric circuit provides a path for theflow of electric current. In general, the term magnetic circuit applies to any closedpath in space, but in theanalysis of electro-mechanical and electronic system this term is specifically used for circuits containing a major portion of ferromagnetic materials. The study of magnetic circuit concepts is essential in the design, analysis and application of electromagnetic devices like transformers, rotating machines, electromagnetic relays etc.

MagnetomotiveForce(M.M.F):

Flux is produced round any current – carrying coil. In order to produce the required flux density, the coil should have the correct number of turns. The product of the current and the number of turns is defined as the coil magneto motive force (m.m.f).

IfI=Currentthroughthecoil(A) N

=Numberof turnsin thecoil.

Magnetomotiveforce=Currentxturns So

M.M.F = I X N

The unit of M.M.F. is ampere–turn (AT) but itis taken as Ampere(A) since N has no dimensions.

MagneticFieldIntensity

Magnetic Field Intensityis defined as the magneto-motive force per unit lengthof the magnetic flux path. Its symbol is H.

$$\text{MagneticfieldIntensity(H)} = \frac{\text{Magnetomotiveforce}}{\text{Meanlengthofthemagneticpath}}$$

- $H = \frac{F}{l} = \frac{I.N.}{l}$ A/m

Where *l* is the mean length of the magnetic circuit in meters. Magnetic field intensity is also called magnetic field strength or magnetizing force.

Permeability:-

Every substance possesses a certain power of conducting magnetic lines of force. For example, iron is better conductor for magnetic lines of force thanair(vaccum).Permeabilityofamaterial(μ)isitsconductingpowerfor magnetic lines of force. It is the ratio of theflux density. (B) Producedina material to the magnetic filed strength (H) i.e. $\mu = B/H$

Reluctance:

Reluctance (s) is akin to resistance (which limits the electric Current). Flux in a magnetic circuit is limited by reluctance. Thus reluctance(s) is a measure of the opposition offered by a magnetic circuit to the setting up of the flux.

Reluctanceistheratioofmagnetomotiveforcetotheflux.Thus

$$S = \text{Mmf}/\phi$$

Itsunitisampereturnsperwebber(orAT/wb)

Permeance:-

Thereciprocalofreluctanceiscalledthepermeance(symbolA).

Permeance (A) = 1/S wb/AT

Turn T has no unit.

Hencepermeanceisexpressedinwb/AorHenerys(H).

ElectricFieldversusMagenticField.

<u>Similarities</u>

	ElectricField		MagneticField
1)	FlowofCurrent(I)	1)	Flowofflux(∅)
2)	Emfisthecauseof flow of current	2)	MMfisthecauseof flow of flux
3)	Resistanceoffered to the flow of Current, is called resistance (R)	3)	Resistanceofferedto the flow of flux, is called reluctance (S)
4)	Conductance $(\sigma)=\frac{1}{R}$	4)	Permitivity(μ)= $\frac{1}{S}$
5)	Current density is amperespersquare meter.	5)	Fluxdensityisnumber of lines per square meter.
6)	Current (I) $= \frac{EMF}{R}$	6)	Flux(∅)= $\frac{MMF}{S}$

<u>Dissimilarities</u>

1)	Currentactuallyflows in an electric Circuit.	1)	Fluxdoesnotactually flow in a magnetic circuit.
2)	Energy is needed as longascurrentflows	2)	Energy is initially needed to create the magneticflux,butnot

			tomaintainit.
3)	Conductance is constant and independentofcurrent strengthataparticular temperature.	3)	Permeability (or magnetic conductance) dependsonthetotal flux for a particular temperature.

B.H.Curve:

Place a piece of an unmagnetised iron bar AB within the field of a solenoid to magnetise it. The field H produced by the solenoid, is called magnetising field, whose value can be altered (increased or decreased) by changing (increasing or decreasing) the current through the solenoid. If we increase slowly the value of magnetic field (H) from zero to maximum value,the value of flux density (B) varies along 1 to 2 as shown in the figure and the magnetic materials (i.e iron bar) finally attains the maximum value of flux density (Bm) at point 2 and thus becomes magnetically saturated.

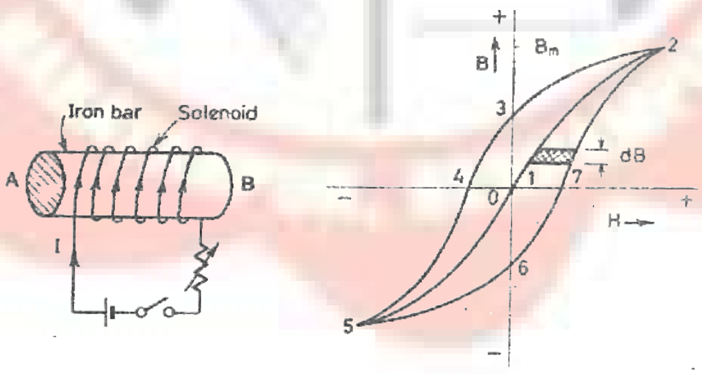


Fig. 2.1

Now if value of H is decreased slowly (by decreasing the current in the solenoid) the corresponding value of flux density (B) does not decreases along 2-1 but decreases some what less rapidly along 2 to 3. Consequently during the reversal of magnetization, the value of B is not zero, but is '13' at H= 0. In other wards, during the period of removal of magnetization force (H), the iron bar is not completely demagnetized.

In order todemagnetise the iron bar completely, we have to supply the demagnetisastion force (H) in the opposite direction (i.e. by reserving the direction of current in the solenoid). The value of B is reduced to zero at point4, when H='14'. This value of H required to clear off the residual magnetisation, is known as coercive force i.e. the tenacity with which the material holds to its magnetism.

If after obtaining zero value of magnetism, the value of H is made more negative, the iron bar again reaches, finally a state of magnetic saturation at the point 5, which represents negative saturation. Now ifthe value of H isincreased from negative saturation (='45') to positive saturation (= '12') a curve '5,6,7,2' is obtained. The closed loop "2,3,4,5,6,7,2" thus represents one complete cycle of magnetisation and is known as hysteresis loop.

NETWORKANALYSIS

Differentterms aredefinedbelow:

1. **Circuit:**Acircuitisaclosedconductingpaththroughwhichanelectriccurrenteither
. flow orisintendedflow

2. **Network:** Acombinationofvariouselectricelements,connectedinany manner. Whatsoever, is called an electric network

3. **Node:**itisanequipotentialpointatwhichtwoormorecircuitelements arejoined.

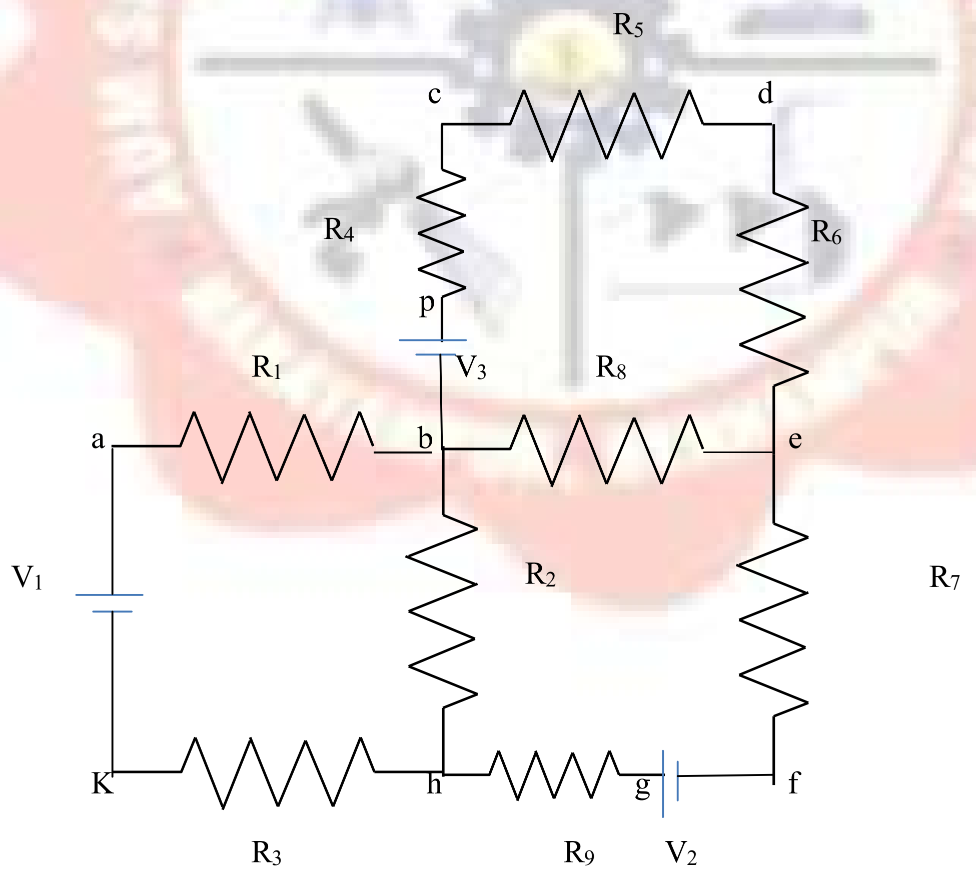
4. **Junction:**itisthat pointofanetwork where threeormorecircuitelementsarejoined.

5. **Branch:**itisapartofanetworkwhichliesbetweenjunctionpoints.

6. **Loop:** Itisaclosedpath inacircuitinwhichnoelementor nodeisaccountedmorethan once.

7. **Mesh:**Itisaloopthatcontainsnootherloopwithinit.

Example 3.1 In this circuit configuration of figure 3.1, obtain the no. of i) circuit elements ii) nodes iii) junction points iv) branches and v) meshes.



Solution:i)no.of circuitelements=12(9 resistors+3voltagesources)

ii) no.ofnodes=10(a, b,c,d, e,f,g, h,k,p)

iii) no. ofjunctionpoints=3(b,e,h)

iv) no.ofbranches=5(bcde,be,bh,befgh,bakh)

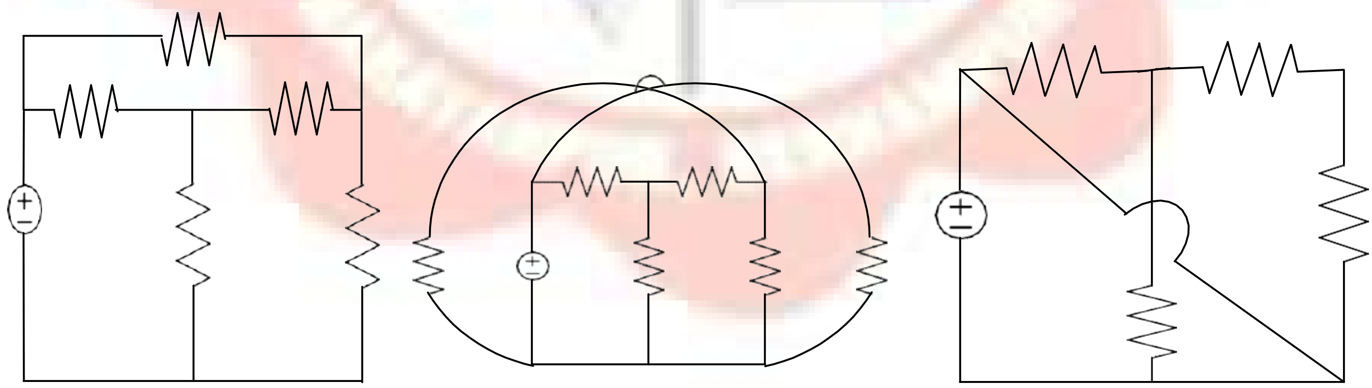
v) no.ofmeshes=3(abhk,bcde, befh)

MESH ANALYSIS

Mesh and nodal analysis are two basic important techniques used in finding solutions for anetwork.Thesuitabilityofeither meshornodalanalysistoaparticular problemdepends mainly on the number of voltage sources or current sources .If a network has a large number of voltage sources, it is useful to use mesh analysis; as this analysis requires that all the sources in a circuit be voltage sources. Therefore, if there are any current sources in a circuit they areto beconvertedinto equivalentvoltagesources,if, onthe other hand, thenetworkhas more current sources,nodal analysis is more useful.

Mesh analysis is applicable only for planar networks. For non-planar circuitsmesh analysis is not applicable .A circuit is said to be planar, if it can be drawn on a plane surface without crossovers. A non-planar circuit cannot be drawn on a plane surface without a crossover.

Figure 3.2 (a) is a planar circuit. Figure 3.2 (b) is a non-planar circuit and fig. 3.2 (c) is a planar circuit which looks like a non-planar circuit. It has already been discussed that a loopis a closed path. Amesh is definedasa loop which does not contain any other loopswithin it. To apply mesh analysis, our first step is to check whether the circuit is planar or not and the second is to select mesh currents. Finally, writingKirchhoff's voltage law equations in terms of unknowns and solving them leads to the final solution.



(a) (b) (c)

Figure 3.2

Observation of the Fig.3.2 indicates that there are two loops abefa,andbcdeb in the network.LetusassumeloopcurrentsI$_1$andI$_2$withdirectionsasindicatedinthefigure.

Considering the loop abefa alone, we observe thatcurrent $I_1$is passing through R_1, and (I_1-I_2) is passing through R_2. By applying Kirchhoff's voltage law, we can write

$$V_s=I_1R_1+R_2(I_1-I_2) \quad (3.1)$$

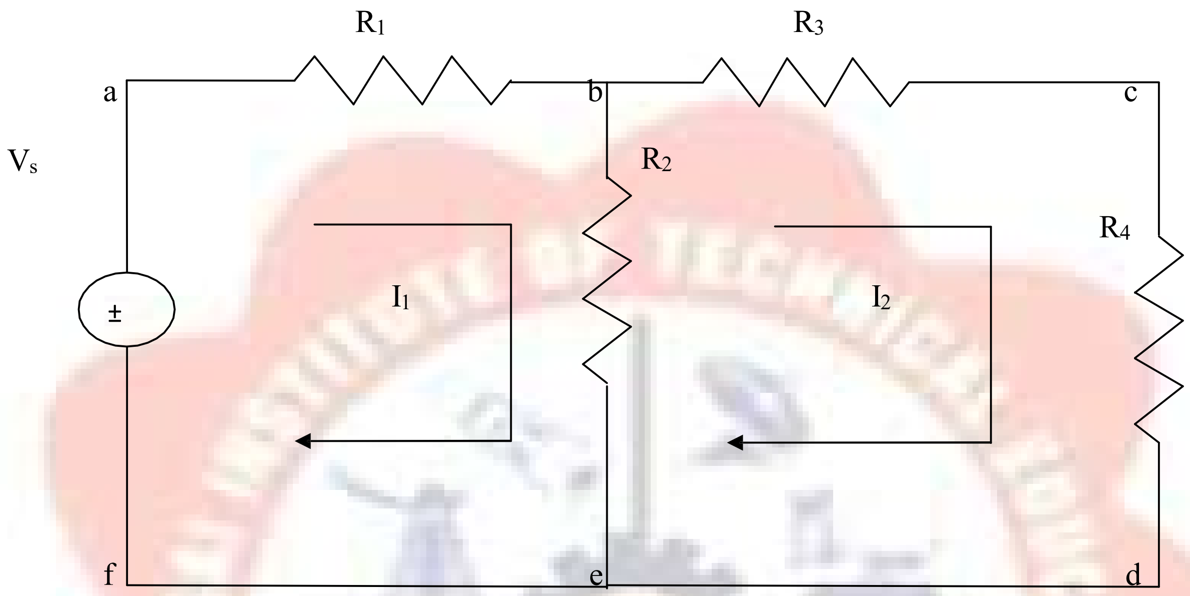


Figure3.3

Similarly, if we consider the second mesh bcdeb, the current $I_2$is passing through $R_3$and R_4,and (I_2-I_1) is passing through R_2.By applying Kirchhoff's voltage law around the second mesh, we have

$$R_2(I_2-I_1)+R_3I_2+R_4I_2=0 \quad (3.2)$$

Byrearrangingtheaboveequations,thecorrespondingmeshcurrentequationsare

$$I_1(R_1+R_2) - I_2R_2 =V_s$$

$$-I_1R_2+(R_2+R_3+R_4)I_2=0 \quad (3.3)$$

By solving the above equations, we can find the currents $I_1$and I_2,.If we observe Fig.3.3, thecircuit consists offive branches and four nodes, includingthe reference node.The number of mesh currents is equal to the number of mesh equations.

And the number of equations=branches-(nodes-1).in Fig.3.3, the required number of mesh current would be 5-(4-1)=2.

IngeneralwehaveBnumberofbranchesandNnumberofnodesincludingthe reference node than number of linearly independent mesh equations M=B-(N-1).

Example3.2 Writethemesh

currentequationsinthecircuit shown

infig3.4anddetermine thecurrents.

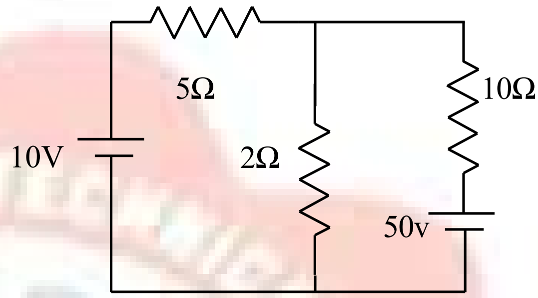


Figure3.4

Solution: Assume two mesh currents in the direction as indicated in fig. 3.5.Themesh currentequationsare

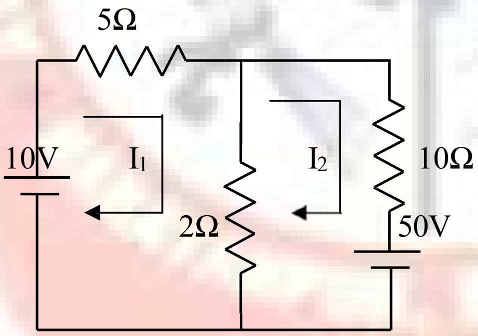


Figure3.5

$5I_1+2(I_1-I_2)=10$

$101_2+2(1_2-1_1)+50=0$ (3.4)

Wecanrearrangetheaboveequationsas $7I_1$

$-2I_2=10$

$-2I_1+12I_2=-50$ (3.5)

Bysolvingtheabove equations,wehave$I_1=0.25A$,and$I_2=-4.125$

Here the current in the second mesh I_2, is negative; that is the actual current I_2 flows opposite to the assumed direction of current in the circuit of fig .3.5.

Example3.3 Determine the mesh currentI_1inthecircuitshowninfig.3.6.

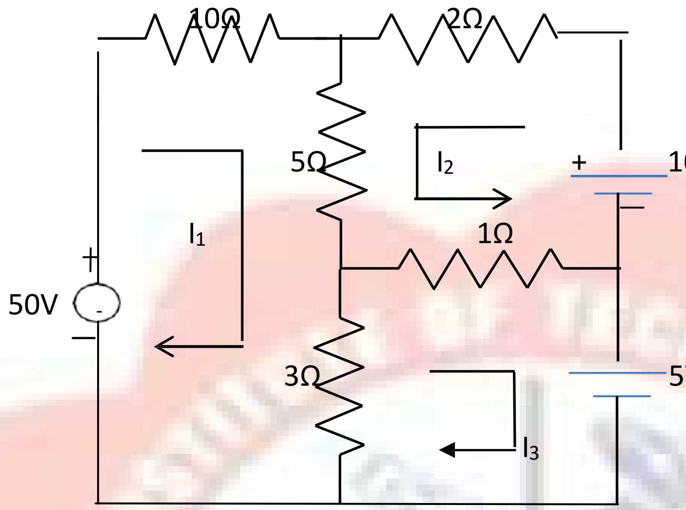


Figure3.6

Solution: From the circuit, we can from the following three mesh equations

$$10I_1+5(I_1+I_2)+3(I_1-I_3)=50 \tag{3.6}$$

$$2I_2+5(I_2+I_1)+1(I_2+I_3)= 10 \tag{3.7}$$

$$3(I_3-I_1)+1(I_3+I_2)=-5 \tag{3.8}$$

Rearrangingtheaboveequationswe get

$$18I_1+5I_2-3I_3=50 \tag{3.9}$$

$$5I_1+8I_2+I_3=10 \tag{3.10}$$

$$-3I_1+I_2+4I_3=-5 \tag{3.11}$$

Accordingtothe Cramer'srule

$$I_1=\frac{\begin{vmatrix}50 & 5 & -3\\ 10 & 8 & 1\\ -5 & 1 & 4\end{vmatrix}}{\begin{vmatrix}18 & 5 & -3\\ 5 & 8 & 1\\ -3 & 1 & 4\end{vmatrix}}=\frac{1175}{356}$$

OrI_1=3.3ASimilarly,

$$I_2=\frac{\begin{vmatrix}18 & 50 & -3\\ 5 & 10 & 1\\ -3 & -5 & 4\end{vmatrix}}{\begin{vmatrix}18 & 5 & -3\\ 5 & 8 & 1\\ -3 & 1 & 4\end{vmatrix}}=\frac{-355}{356}$$

OrI_2=-0.997A (3.12)

$$I_3=\frac{\begin{vmatrix}18 & 5 & 50\\ 5 & 8 & 10\\ -3 & 1 & -5\end{vmatrix}}{\begin{vmatrix}18 & 5 & -3\\ 5 & 8 & 1\\ -3 & 1 & 4\end{vmatrix}}=\frac{525}{356}$$

Or I_3=1.47A (3.13)

$\therefore I_1$=3.3A, I_2=-0.997A, I_3=1.47A

MESH EQUATIONS BY INSPECTION METHOD The mesh equations for a general planar network can be writtenby inspection without going through the detailed steps. Consider a three mesh networks as shown in figure 3.7

The loop equation are $I_1R_1+ R_2(I_1-I_2) \quad =V_1$

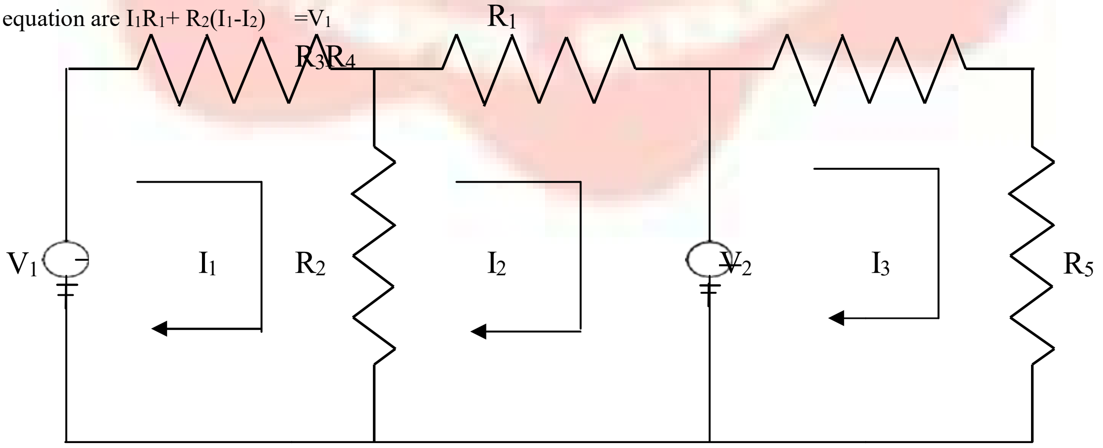


Figure3.7

$R_2(I_2-I_1)+I_2R_3=-V_2$ 3.14

$R_4I_3+R_5I_3=V_2$ 3.15

Reordering theaboveequations,wehave

$(R_1+R_2)I_1-R_2I_2=V_1$ 3.16

$-R_2I_1+(R_2+R_3)I_2=-V_2$ 3.17

$(R_4+R_5)I_3=V_2$ 3.18

Thegeneralmeshequationsforthreemeshresistivenetworkcanbewrittenas $R_{11}I_1\pm$

$R_{12}I_2\pm R_{13}I_3= V_a$ 3.19

$\pm R_{21}I_1+R_{22}I_2\pm R_{23}I_3=V_b$ 3.20

$\pm R_{31}I_1\pm R_{32}I_2+R_{33}I_3=V_c$ 3.21

By comparing the equations 3.16, 3.17 and 3.18 with equations 3.19, 3.20 and 3.21respectively, the following observations can be taken into account.

1. Theself-resistanceineachmesh
2. Themutualresistancesbetweenall pairsofmeshesand
3. Thealgebraic sumofthevoltagesineachmesh.

The self-resistance of loop 1, $R_{11}=R_1+R_2$, is the sum of the resistances through which I_1passes.

The mutual resistance of loop 1, $R_{12}= -R_2$, is the sum of the resistances common to loop currents $I_1$and I_2.If the directions of the currents passing through the common resistances are the same, the mutual resistance will have a positive sign; and if the directions of the currents passing through the common resistance are opposite then the mutual resistance will have a negative sign.

$V_a=V_1$is the voltage which drives the loop 1. Here the positive sign is used if the direction of the currents is the same as the direction of the source. If the current direction is opposite to the direction of the source, then the negative sign is used.

Similarly $R_{22}=R_2+R_3$and $R_{33}=R_4+R_5$are the self-resistances of loops 2 and 3 respectively. The mutual resistances $R_{13}=0$, $R_{21}= -R_2$, $R_{23}=0$, $R_{31}=0$, $R_{32}=0$ are the sums of the resistances common to the mesh currents indicated in their subscripts.

$V_b=-V_2$,$V_c= V_2$arethesumofthevoltagesdrivingtheirrespective loops.

Example 3.4write themeshequationforthecircuitshown infig.3.8

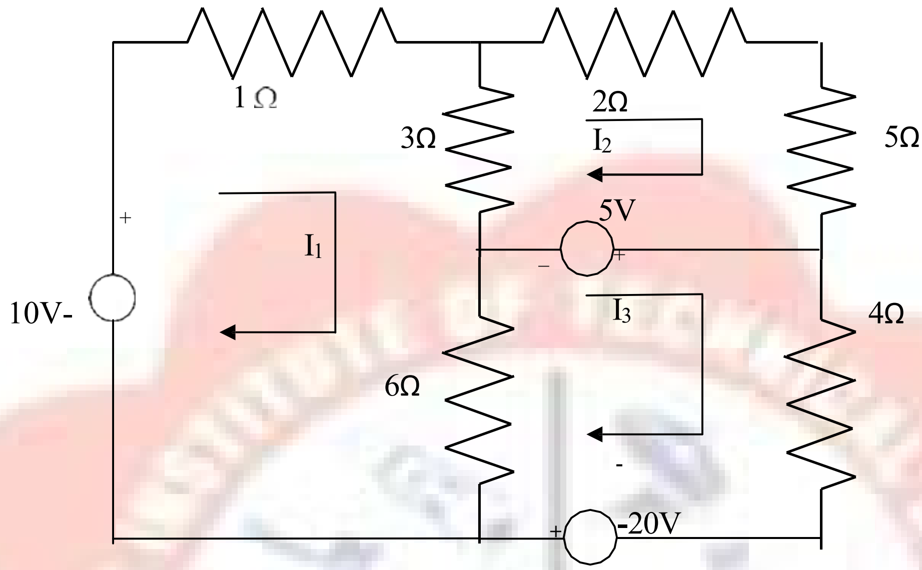


Figure3.8

Solution:thegeneralequationforthreemeshequationare

$R_{11}I_1 \pm R_{12}I_2 \pm R_{13}I_3 = V_a$ (3.22)

$\pm R_{21}I_1 + R_{22}I_2 \pm R_{23}I_3 = V_b$ (3.23)

$\pm R_{31}I_1 \pm R_{32}I_2 + R_{33}I_3 = V_c$ (3.24)

Considerequation3.22

R_{11}=selfresistanceofloop1=(1Ω+ 3Ω+6Ω)=10 Ω

R_{12}=themutualresistancecommontoloop1andloop 2 =-3 Ω

Herethenegativesignindicatesthat thecurrentsareinoppositedirection.R_{13}= the mutual resistance common to loop 1 & 3= -6 Ω

V_a=+10V,the voltage the drivingthe loop1.

HerehepositivesignindicatestheloopcurrentI$_1$isinthesamedirectionasthe source element.

Thereforeequation3.22canbe writtenas

$10I_1-3I_2-6I_3=10V$ (3.25)

ConsiderEq.3.23

R_{21}=themutualresistancecommontoloop1andloop 2 =-3 Ω

R_{22}= self resistance of loop 2=(3Ω+ 2 Ω +5 Ω) =10 Ω
R_{23}=0,thereisnocommonresistancebetweenloop2and3. V_b = -5 V, the voltage driving the loop 2.

ThereforeEq. 3.23canbewrittenas

$-3I_1+10I_2=-5V$ (3.26)

ConsiderEq.3.24

R_{31}=themutualresistancecommontoloop1andloop3= -6Ω R_{32}= the mutual resistance common to loop 3 and loop 2 = 0 R_{33}= self resistance of loop 3=(6Ω+ 4 Ω) =10 Ω
V_c=thealgebraicsumofthevoltage drivingloop3
=(5 V+20V)=25V (3.27)

Therefore,Eq3.24canbewrittenas-$6I_1+10I_3$= 25V
$-6I_1-3I_2-6I_3=10V$
$-3I_1+10I_2=-5V$
$-6I_1+10I_3=25V$

SUPERMESHANALYSIS

Suppose any of the branches in thenetwork has acurrent source, then it isslightly difficultto apply mesh analysis straight forward because first we should assume an unknown voltage across the current source, writing mesh equation as before, and then relate the source current to theassignedmesh currents. Thisisgenerally adifficult approach.Onway to overcomethis difficulty is by applying the supermesh technique. Here we have to choose the kind of supermesh. A supermesh is constituted by two adjacent loops that have a common current source. As an example, consider the network shown in the figure 3.9.

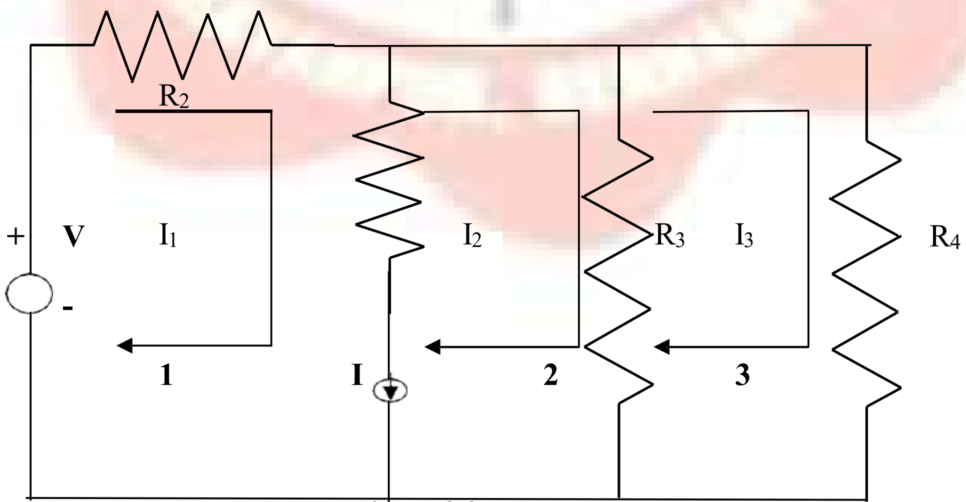


Figure3.9

Herethecurrent sourceI isinthecommon boundaryfor thetwomeshes1and2.Thiscurrent source creates a supermesh, which is nothing but a combination of meshes 1 and 2.

$R_1I_1 + R_3(I_2-I_3)=V$

Or $R_1I_1+R_3I_2-R_4I_3=V$

Consideringmesh3,wehave

$R_3(I_3-I_2)+ R_4I_3=0$

Finally thecurrentI fromcurrent sourceisequaltothedifferencebetweentwomeshcurrents i.e.

$I_1-I_2=I$

wehavethusformedthreemeshequationswhichwecansolveforthethreeunknown currents in the network.

Example3.5.Determinethecurrentinthe5Ωresistorinthenetworkgiveninfig.3.10

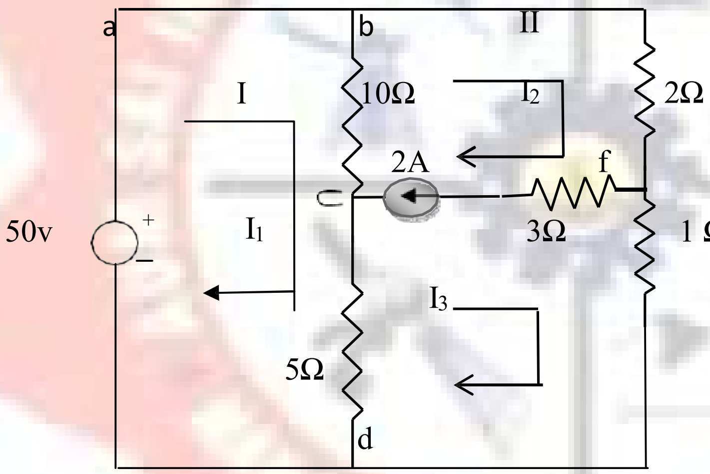


Figure3.10

Solution:-Fromthefirstmesh,i.e.abcda,wehave

$50=10(I_1-I_2)+5(I_1-I_3)$

Or $15I_1-10I_2-5I_3=50$ (3.28)

Fromthesecondandthirdmeshes.wecan form asupermesh

$10(I_2-I_1)+2I_2+I_3+5(I_3-I_1)=0$

Or $-15I_1+12I_2+6I_3=0$ (3.29)

Thecurrentsourceisequal tothedifferencebetween IIand IIImesh currents

i.e.I_2-I_3= 2A (3.30)

Solving3.28.,3.29and3.30.wehave

I_1=19.99A,I_2=17.33A,andI_3=15.33A

Thecurrentinthe5Ωresistor =I_1-I_3

=19.99-15.33=4.66A

The currentinthe5Ωresistoris4.66A.

Example 3.6. Write the mesh equations for the circuit shown in fig. 3.11 and determine the currents, I_1, $I_2$and I_3.

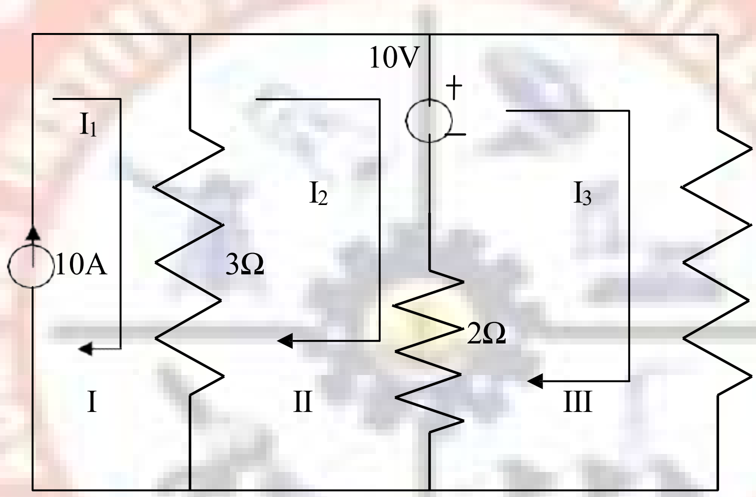


Figure3.11

Solution ;In fig 3.11, the current source lies on the perimeter of the circuit, and thefirst mesh is ignored. Kirchhoff‘s voltage law is applied only for second and third meshes .

Fromthesecondmesh,wehave

$3(I_2-I_1)+2(I_2-I_3)+10 =0$

Or $-3I_1+5I_2-2I_3=-10$ (3.31)

Fromthethirdmesh,wehave I_3 +

$2 (I_3 -I_2) =10$

Or $-2I_2+3I_3=10$ (3.32)

From the first mesh, $I_1=10A$ (3.33)

From the abovethree equations, we get

$I_1=10A$, $I_2=7.27$, $I_3=8.18A$

NODALANALYSIS

In the chapter I we discussed simple circuits containing only two nodes, including the reference node. In general, in a N node circuit, one of the nodes is chosen as the reference or datum node, then it is possible to write N -1nodal equations by assuming N-1 node voltages.For example,a10 node circuit requires nine unknown voltages and nine equations. Each node in a circuit can be assigned a number or a letter. The node voltage is the voltage of a given node with respect to oneparticularnode, called thereferencenode, which weassumeat zero potential. In thecircuit shown in fig. 3.12, node 3 is assumed as theReference node. The voltage at node 1 is the voltage at that node with respect to node 3. Similarly, the voltage at node 2 is the voltage at that node with respect to node 3. Applying Kirchhoff's current law at node 1, the current entering is the current leaving (See Fig.3.13)

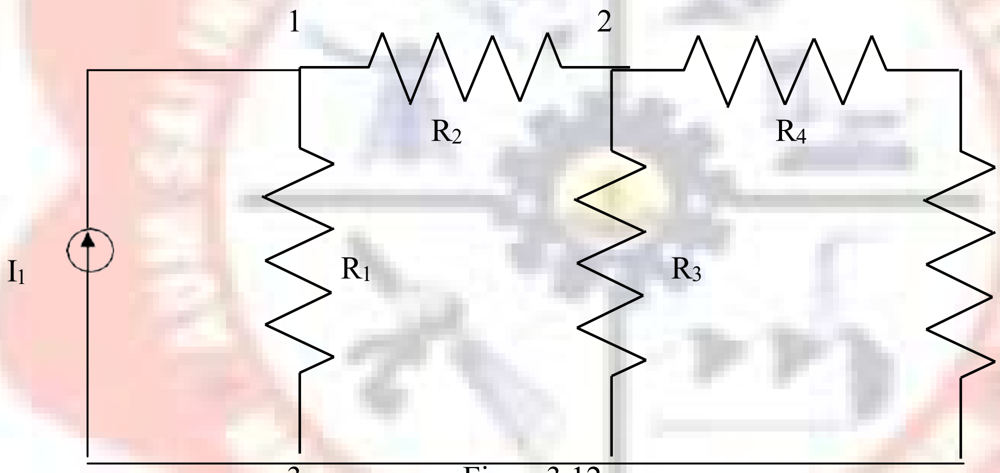


Figure3.12

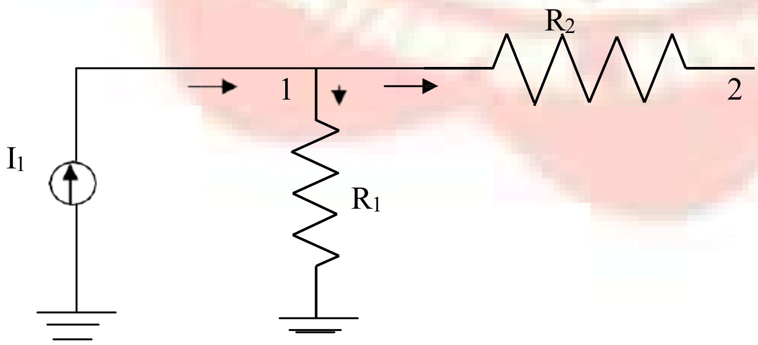


Figure3.13

$I_1= V_1/R_1 + (V_1-V_2)/R_2$

Where V_1 and V_2 arethevoltagesatnode1and2,respectively.Similarly,atnode 2.the currententeringisequaltothecurrentleaving asshown infig.3.14

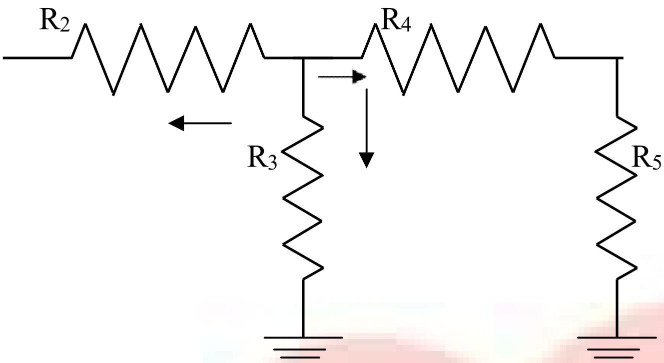


Figure3.14

$(V_2-V_1)/R_2+V_2/R_3+V_2/(R_4+R_5)=0$

Rearrangingtheaboveequations,wehave

$V_1[1/R_1+1/R_2]-V_2(1/R_2)= I_1$

$-V_1(1/R_2)+V_2[1/R_2+1/R_3+1/(R_4+R_5)]=0$

Fromthe above equationswecanfindthe voltagesateachnode.

Example3.7 Determinethevoltagesateachnodeforthecircuitshowninfig3.15

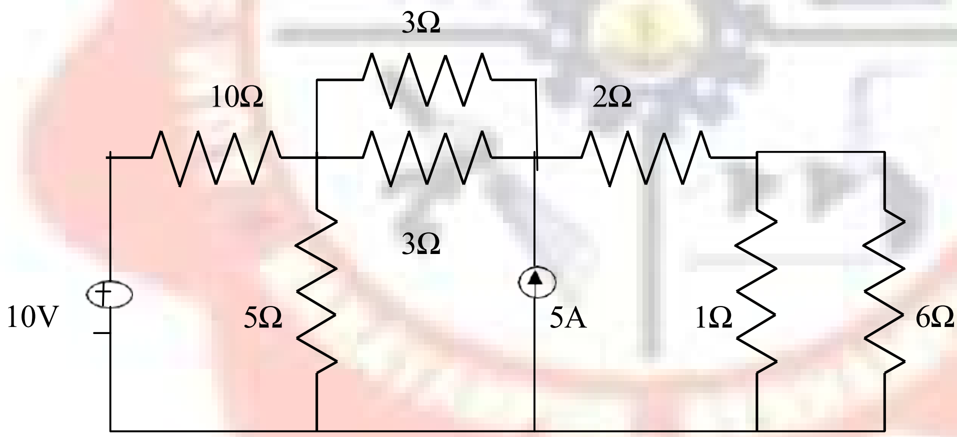


Figure3.15

Solution: Atnode1,assumingthatallcurrentsareleaving,wehave $(V_1-10)/10 + (V_1-V_2)/3 +V_1/5 + (V_1-V_2)/3 =0$

Or $V_1[1/10+1/3+1/5+1/3]-V_2[1/3+ 1/3]=1$

$0.96V_1-0.66V_2= 1$ (3.36)

At node 2,assuming that all currentsare leaving except the current from current source,wehave

$(V_2-V_1)/3+(V_2-V_1)/3+(V_2-V_3)/2=5$

$-V_1[2/3]+V_2[1/3+1/3+1/2]-V_3(1/2)=5$

$-0.66V_1+1.16V_2-0.5V_3= 5$ (3.37)

At node 3 assuming all currents are leaving, we have $(V_3-V_2)/2 + V_3/1 + V_3/6 = 0$

$$-0.5V_2+1.66V_3=0 \qquad (3.38)$$

Applying Cramer's rule we get

$$V_1=\frac{\begin{vmatrix} 1 & -0.66 & 0 \\ 5 & 1.16 & -0.5 \\ 0 & -0.5 & 1.66 \end{vmatrix}}{\begin{vmatrix} 0.96 & -0.66 & 0 \\ -0.66 & 1.16 & -0.5 \\ 0 & -0.5 & 1.66 \end{vmatrix}}=\frac{7.154}{0.887}=8.06$$

Similarly,

$$V_2=\frac{\begin{vmatrix} 0.96 & 1 & 0 \\ -0.66 & 5 & -0.5 \\ 0 & 0 & 1.66 \end{vmatrix}}{\begin{vmatrix} 0.96 & -0.66 & 0 \\ -0.66 & 1.16 & -0.5 \\ 0 & -0.5 & 1.66 \end{vmatrix}}=\frac{9.06}{0.887}=10.2$$

$$V_3=\frac{\begin{vmatrix} 0.96 & -0.66 & 1 \\ -0.66 & 1.16 & 5 \\ 0 & -0.5 & 0 \end{vmatrix}}{\begin{vmatrix} 0.96 & -0.66 & 0 \\ -0.66 & 1.16 & -0.5 \\ 0 & -0.5 & 1.66 \end{vmatrix}}=\frac{2.73}{0.887}=3.07$$

NODAL EQUATIONS BY INSPECTION METHOD The nodal equations for a general planar network can also be written by inspection without going through the detailed steps. Consider a three node resistive network, including the reference node, as shown in fig 3.16

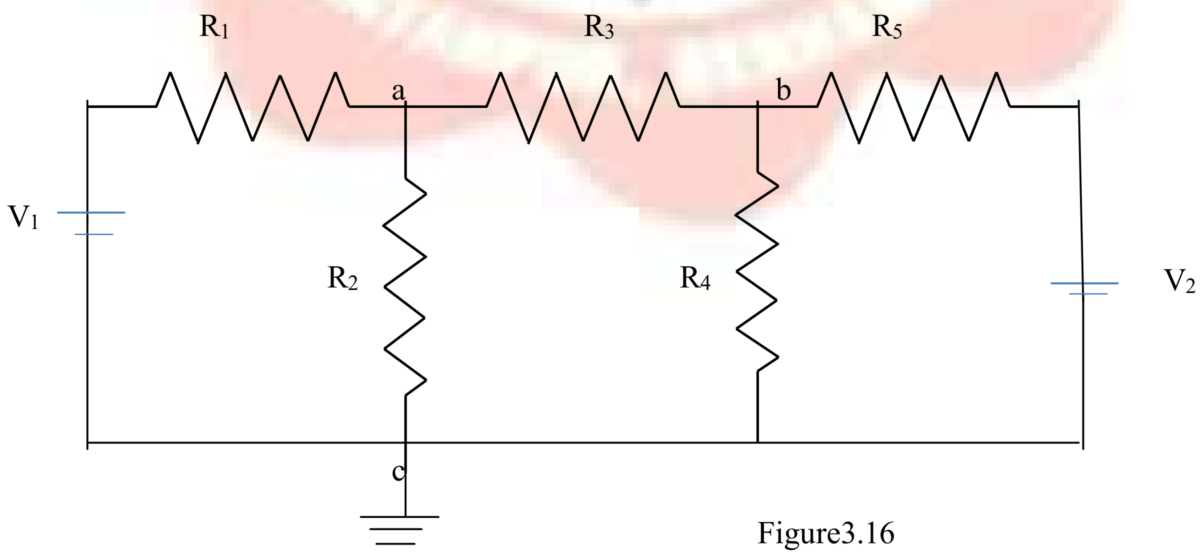


Figure 3.16

Infig. 3.16thepointsaandbaretheactualnodesandcisthereferencenode. Now consider the nodes a and b separately as shown in fig 3.17(a) and (b)

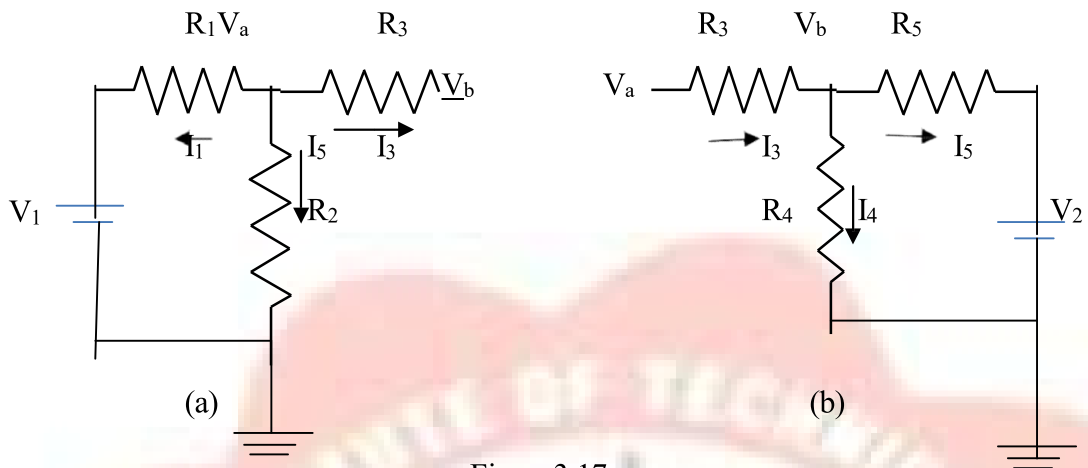


Figure3.17

Infig3.17(a),accordingtoKirchhoff'scurrentlawwehave

$I_1+I_2+I_3=0$

$(V_a-V_1)/R_1+V_a/R_2+(V_a-V_b)/R_3=0$ (3.39)

Infig3.17(b),ifweapplyKirchhoff'scurrentlaw

$I_4+I_5=I_3$

$\therefore (V_b-V_a)/R_3+ V_b/R_4+(V_b-V_2)/R_5=0$ (3.40)

Rearrangingtheaboveequationswe get

$(1/R_1+1/R_2+1/R_3)V_a-(1/R_3)V_b=(1/R_1)V_1$ (3.41)

$(-1/R_3)V_a+(1/R_3+1/R_4+1/R_5)V_b=V_2/R_5$ (3.42)

In general, theabove equationcanbe writtenas

$G_{aa}V_a+ G_{ab}V_b=I_1$ (3.43)

$G_{ba}V_a+ G_{bb}V_b=I_2$ (3.44)

By comparing Eqs 3.41,3.42 and Eqs 3.43, 3.44 we have the self conductance at node a, $G_{aa}=(1/R_1+ 1/R_2+ 1/R_3)$ is the sum of the conductances connected to node a. Similarly, $G_{bb}= (1/R_3+ 1/R_4+1/R_5)$ is the sum of the conductances connected to node b. $G_{ab}=(-1/R_3)$ is the sum of the mutual conductances connected to node *a* and node *b*. Here all the mutual conductances have negative signs. Similarly, $G_{ba}= (-1/R_3)$ is also a mutual conductance connected between nodes *b* and *a*. $I_1$and $I_2$are the sum of the source currents at node *a* and node *b*, respectively. The current which drives into the node has positive sign, while the current that drives away from the node has negative sign.

Example3.8forthecircuitshowninthefigure3.18writethenodeequationsbythe inspection method.

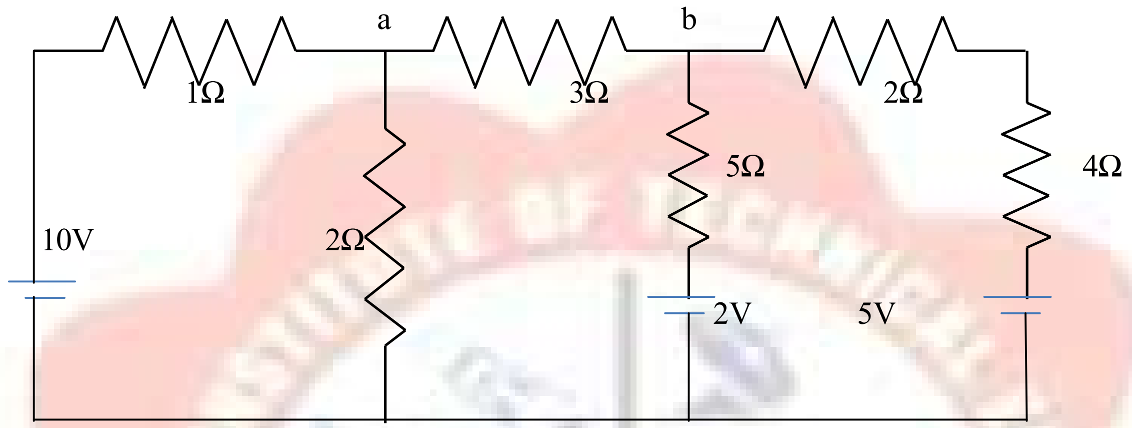


Fig3.18

Solution:-

The generalequationsare

$G_{aa}V_a+G_{ab}V_b=I_1$ (3.45)

$G_{ba}V_a+ G_{bb}V_b=I_2$ (3.46)

Considerequation 3.45

G_{aa}=(1+1/2+1/3)mho.Theself conductanceatnode *a*isthesumoftheconductancesconnected to node *a*.

G_{bb}=(1/6+1/5+1/3)mhotheself conductanceatnode *b*isthesumof conductancesconnected to node *b*.

G_{ab}=-(1/3)mho, themutualconductancesbetweennodes*a*and*b*isthesumof the conductances connected between node *a* and *b*.

SimilarlyG_{ba}=-(1/3),thesumofthemutualconductancesbetweennodes *b*and*a*. I_1=10/1 =10

A, the source current at node *a*,

I_2=(2/5+5/6)=1.23A,thesourcecurrentatnode*b*.

Therefore, the nodal equations are

$1.83V_a-0.33V_b=10$ (3.47)

$-0.33V_a+0.7V_b=1.23$ (3.48)

SUPERNODEANALYSIS

Supposeany of thebranchesin thenetwork hasa voltagesource, thenit isslightly difficult to apply nodal analysis. One way to overcome this difficulty is to apply thesupernode technique. In this method, the two adjacent nodes that are connected by a voltage source are reduced to a single node and then the equations are formed by applying Kirchhoff's current law as usual. This is explained with the help of fig. 3.19

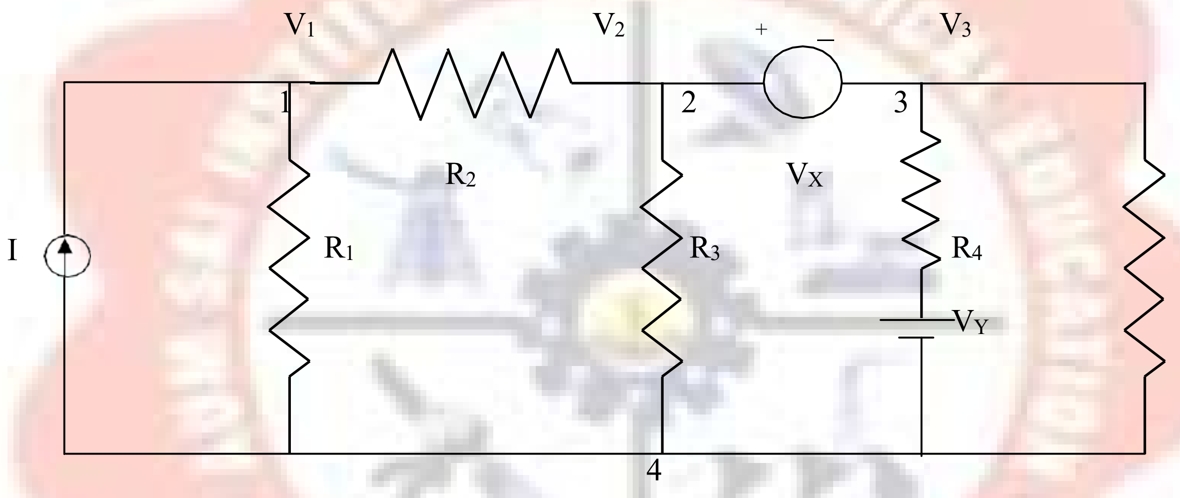


FIG3.19

It isclearfromthefig.3.19,thatnode4isthereference node.ApplyingKirchhoff's current law at node 1, we get

$I=(V_1/R_1)+(V_1-V_2)/R_2$

DuetothepresenceofvoltagesourceV$_\chi$inbetweennodes2and3,itisslightly difficult to find out the current. The supernode technique can be conveniently applied in this case.

Accordingly,we canwritethecombinedequationfornodes2and3as under.

$(V_2-V_1)/R_2+V_2/R_3+(V_3-V_y)/R_4+V_3/R_5=0$

Theotherequationis

$V_2-V_3=V_x$

Fromtheabove threeequations,wecanfindthe threeunknownvoltages.

Example3.9 Determinethecurrentinthe5Ωresistorforthecircuitshowninfig. 3.20

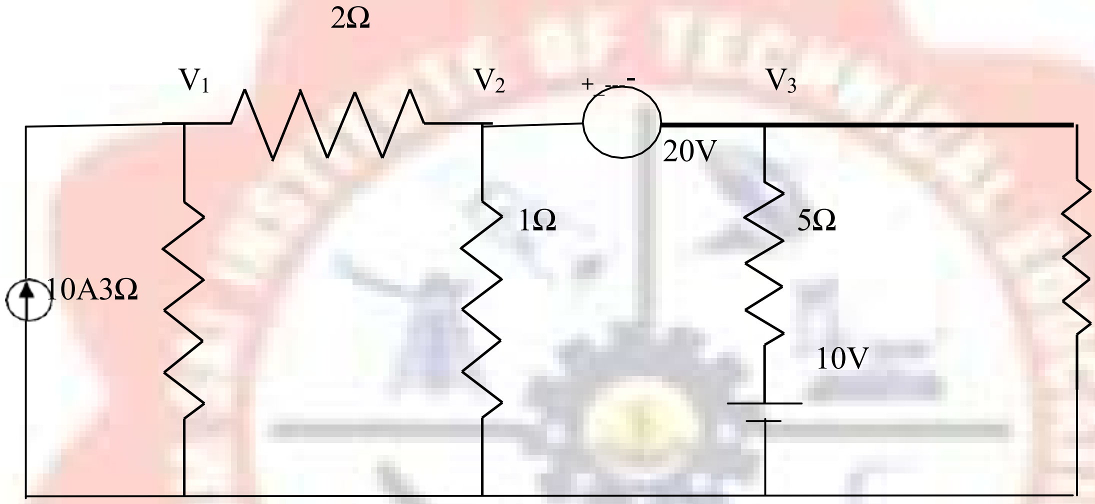


fig.3.20

Solution. Atnode1

$10= V_1/3+(V_1-V_2)/2$

Or $V_1[1/3+1/2]-(V_2/2)-10=0$

$0.83V_1-0.5V_2-10=0$ (3.49)

Atnode2and3,thesupernodeequationis

$(V_2-V_1)/2+ V_2/1+(V_3-10)/5+V_3/2=0$

Or $-V_1/2+V_2[(1/2)+1]+V_3[1/5+1/2]=2$

Or $-0.5V_1+1.5V_2+0.7V_3-2=0$ (2.50)

The voltagebetweennodes2and3isgivenby

$V_2-V_3=20$ (3.51)

The current in 5Ω resistor I_5 =(V_3-

10)/5Solvingequation3.49,3.50and3.51,weobtai

n

V_3 =-8.42V

∴ CurrentI_5=(-8.42-10)/5=-3.68A(currenttowardsnode3)i.ethecurrent flows towards node 3.

SOURCETRANSFORMATIONTECHNIQUE

In solving networkstofind solutions onemay have to deal with energysources. Ithas already been discussed in chapter 1 that basically, energy sources are either voltage sourcesor current sources. Sometimes it is necessary to convert a voltagesource to a current source or vice-versa. Any practical voltage source consists of an ideal voltage source in series with an internal resistance. Similarly, a practical current source consists of an ideal current source in parallel with an internal resistance as shown in figure3.21. $R_v$and $R_i$represent the internal resistances of the voltage source V_s ,and current source I_s ,respectively.

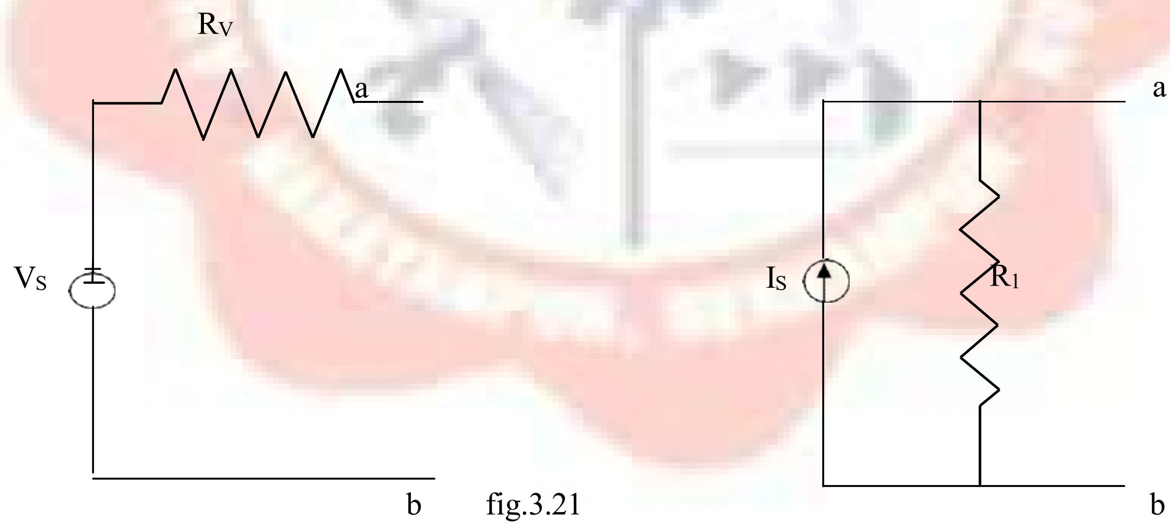
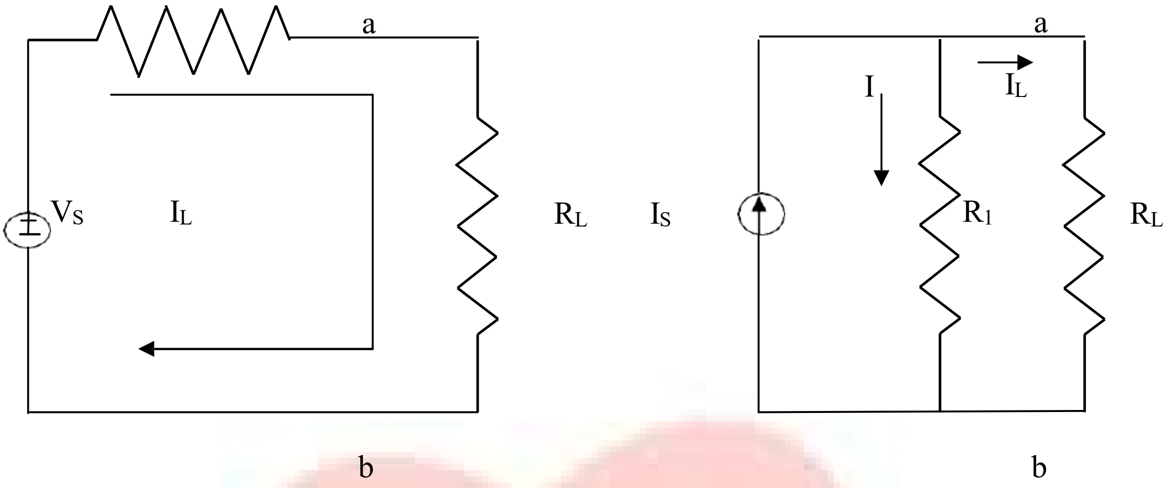


fig.3.21

Any source, be it a current source or a voltage source, drives currentthrough its load resistance,andthemagnitudeofthecurrentdependsonthevalueoftheloadresistance.Fig 3.22representsapracticalvoltagesourceandapracticalcurrentsourceconnectedtothe same load resistance R_L.

R_V



(a) (b)

Figure3.22

Fromfig3.22(a)theloadvoltage canbe calculated by usingKirchhoff'svoltage law as

$V_{ab}=V_s-I_LR_v$

TheopencircuitvoltageV$_{oc}=V_s$

TheshortcircuitcurrentI$_{sc}=\frac{V_s}{R_v}$

from fig3.22(b)

$I_L=I_s-I=I_s-(V_{ab}/R_1)$

TheopencircuitvoltageV$_{oc}=I_sR_1$Th

e short circuit current $I_{sc}=I_s$

The above two sources are said to be equal, if they produce equal amounts of current and voltage when they are connected to identical load resistances. Therefore, by equating the open circuit votages and short circuit currents of the above two sources we obtain

$V_{oc}=I_sR_1=V_sI_{sc}=$

$I_s=V_s/R_v$

It follows that

$R_1=R_v=R_s$; $V_s=I_sR_s$

where $R_s$is the internal resistance of the voltage or current source. Therefore, any practical voltage source, having an ideal voltage $V_s$and internal series resistance $R_s$can be replacedbyacurrentsource$I_s=V_s/R_s$inparallelwithaninternalresistanceR_s.Thereverse tansformation is also possible. Thus, a practical current source in parallel with an internal resistance $R_s$can be replaced by a voltage source $V_s=I_sR_s$in series with an internal resistance R_s.

Example 3.10 Determine the equivalent voltage source for the current source shown in fig 3.23

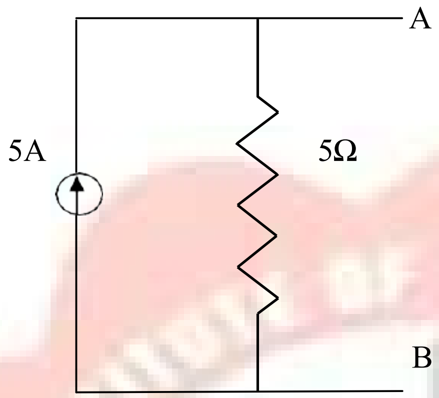


Figure3.23

Solution: ThevoltageacrossterminalsAandBisequalto25V. sincetheinternalresistance for the current source is 5 Ω, the internal resistance of the voltage source is also 5 Ω. The equivalent voltage source is shown in fig. 3.24.

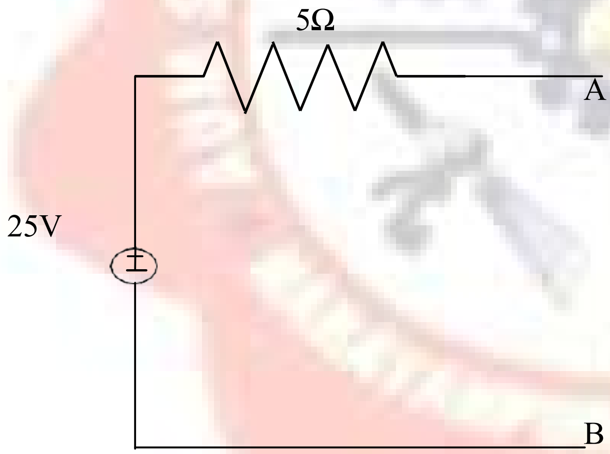
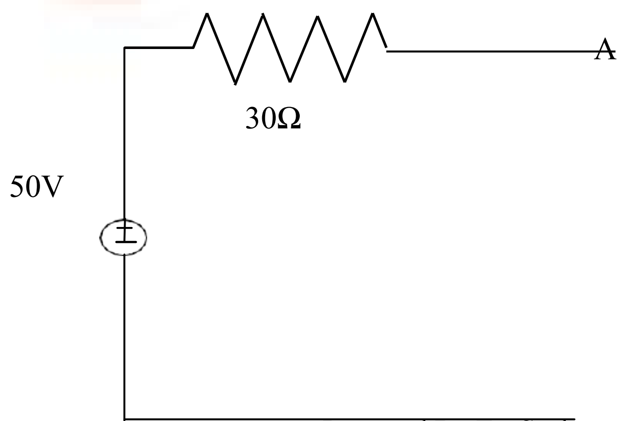


Fig3.24

Example3.11 Determinetheequivalentcurrentsourceforthevoltagesourceshowninfig.3.25



Solution:theshortcircuitcurrentatterminalsAandBisequalto I= 50/30

= 1.66 A

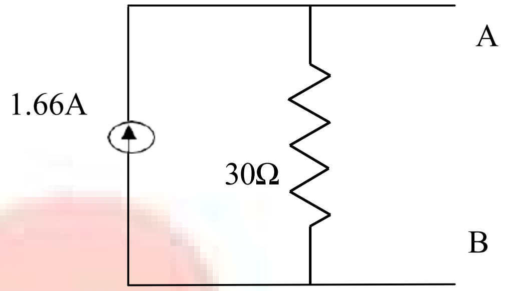


Fig3.26

Sincetheinternalresistanceforthevoltagesourceis30Ω, the internalresistanceof the current source is also 30 Ω. The equivalent current source is shown in fig. 3.26.

NETWORKTHEOREMS

Beforestartthetheoremweshouldknowthebasictermsofthenetwork.

Circuit: Itisthecombinationofelectricalelementsthroughwhichcurrent passes is called circuit.

Network: It is the combination of circuits and elements is called network.

Unilateral:Itisthecircuitwhoseparameterandcharacteristicschangewith change in the direction ofthe supply application.

Bilateral: Itisthecircuitwhoseparameterandcharacteristicsdonotchange with the supply in either side of the network.

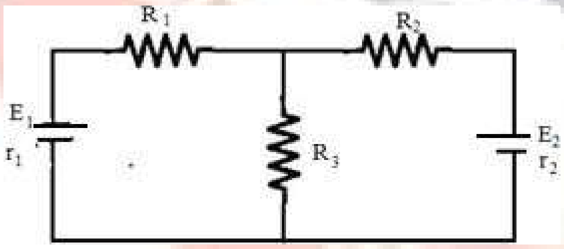
Node: Itistheinterconnectionpointoftwoormorethantwoelementsis called node.

Branch: Itistheinterconnectionpointofthreeormorethanthreeelementsis called branch.

Loop: Itisacompleteclosedpathinacircuitandnoelementornodeistaken more than once.

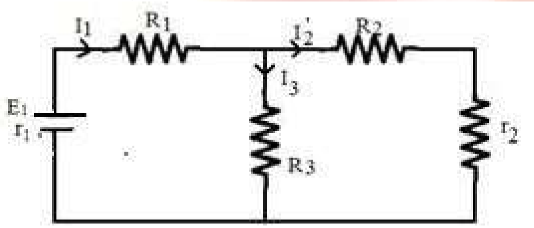
Super-PositionTheorem :

Statement :"It statesthat ina network oflinear resistancescontaining more than one source the current which flows at any point is the sum of all the currents which would flow at that point if each source were considered separatelyand all other sources replaced for time being leaving its internal resistances if any".



Explanation:

ConsideringE_1source



Step1.

R_2&rareinseriesandparallelwithR_3andagainserieswithR_1

$(R_2+r_2)\|R_3$

$= \dfrac{(R_2+r_2)R_3}{R_2+r_2+R_3} = m$ (say)

$Rt_1 = m+R_1+r_1$

$I_1 = \dfrac{E_1}{Rt_1}$

$I_2 = \dfrac{I_1 \times R_3}{R_2+r_2+R_3}$

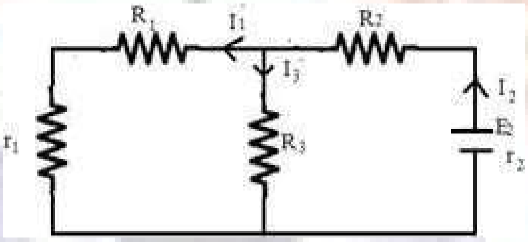
$I_3 = \dfrac{I_1(R_2+r_2)}{R_2+r_2+R_3}$

Step–2

Considering E2 source, R_1 & r_2 are series and R_3 parallel and R_2 in series

$(R_1+r_1)\|R_3$

$= \dfrac{(R_1+r_1)R_3}{R_1+r_1+R_3} = n$ (say)



$Rt_2 = n+R_2+r_2$

$I_2 = \dfrac{E_2}{Rt_2}$

$I_3^{/} = \dfrac{I_2(R_1+r_1)}{R_1+r_1+R_3}$

$I_1^{/} = \dfrac{I_2 \times R_3}{R_1+r_1+R_3}$

Step–3

Current in R_1 branch $= I_1 - I_1^{/}$

Current in R_2 branch $= I_2 - I_2^{/}$

Current in R_3 branch $= I_3 - I_3^{/}$

The direction of the branch current will be in the direction of the greater value current.

Thevenin's Theorem:

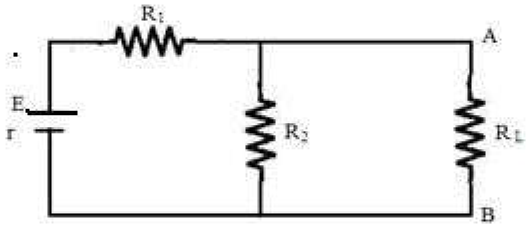
The current flowing through the load resistance R_1 connected across any two terminals A and B of a linear active bilateral network is given by

$I_L = \dfrac{V_{th}}{R_{th}+R_L} = \dfrac{V_{oc}}{R_i+R_L}$

Where $V_{th} = V_{oc}$ is the open. circuit voltage across A and B terminal when R_L is removed.

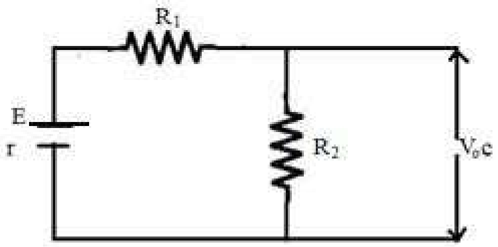
$R_i = R_{th}$ is the internal resistances of the network as viewed back into the open circuit network from terminals A & B with all sources replaced by their internal resistances if any.

Explanation:



<u>Step–1</u> for finding V_{oc}

Remove R_L temporarily to find V_{oc}.

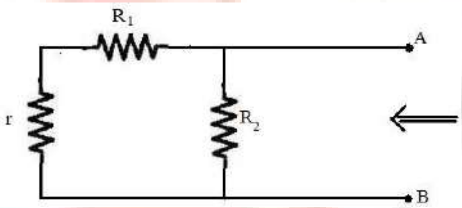


$$I = \frac{E}{R_1 + R_2 + r}$$

$$V_{oc} = IR_2$$

<u>Step–2</u> finding R_{th}

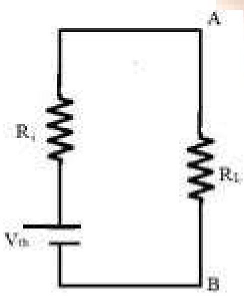
Remove all the sources leaving their internal resistances if any and viewed from open circuit side to find out R_i or R_{th}.



$$R_i = (R_1 + r) \| R_2$$

$$R_i = \frac{(R_1 + r)R_2}{R_1 + r + R_2}$$

<u>Step–3</u>



Connect internal resistances and Thevenin's voltage in series with load resistance R_L.

WhereR_{th}=thevenin resistance

V_{th}=thevenin voltage

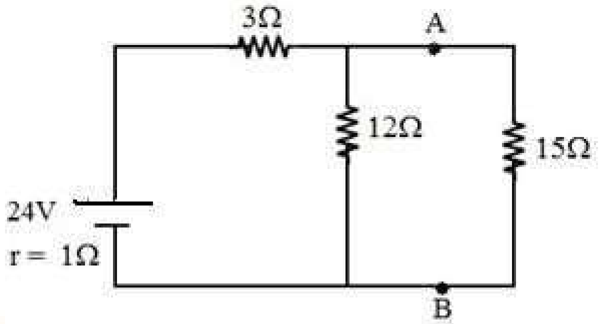
I_{th}=thevenin current

$R_i=(R_1+r)||R_2$

$$I_L = \frac{V_{th}}{R_{th}+R_L} = \frac{V_{oc}}{R_i+R_L}$$

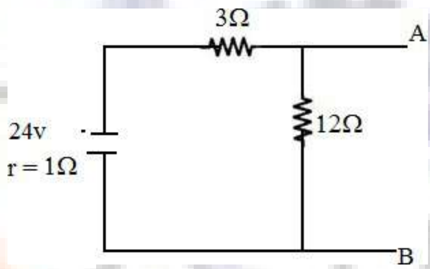
Example 01- Applyingthevenintheoremfindthefollowingfromgiven figure

(i) theCurrent in the load resistance$R_L$of15 Ω



Solution:(i)FindingVoc

→Remove15Ωresistanceandfindthe Voltageacross AandB



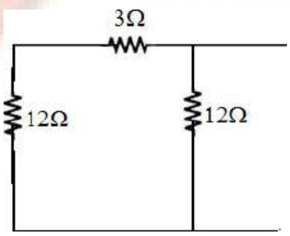
V_{oc}isthevoltageacross12Ωresister

$$V_{oc}=\frac{24\times12}{12+3+1}=18V$$

(ii) FindingR_{th}

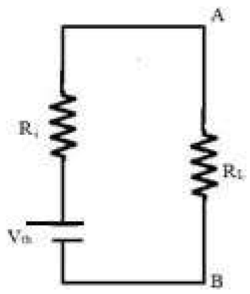
R_{th}iscalculatedfromtheterminalA&Bintothe network.

The 1 Ωresister and 3 Ωin are series and then parallel



R_{th}=3+1 //12

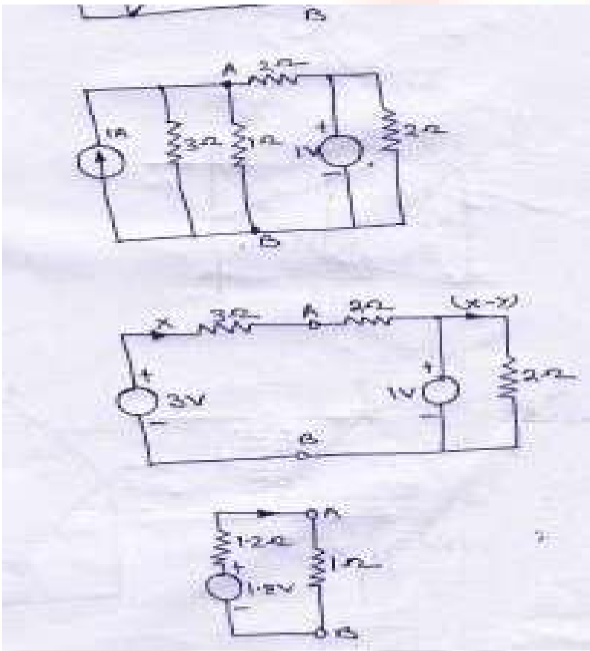
$$=\frac{4\times12}{16}=3\Omega$$



(iii) $I_{th} = \frac{V_{oc}}{R_L + R} = \frac{18}{15+3} = 1A.$

<u>Example02:</u> Determinethecurrentin1Ωresistoracross ABofthenetwork showninfig(a)usingthevenintheorem.

Solution: Thecircuirtcanberedrawnasinfig(b).



fig(a),(b),(c),(d)respectively

Step-1 remove the 1Ω resistor and keeping open circuit .The current source isconverted to the equivalent voltage source as shown in fig (c)

Step-02forfindingtheV_{th}we'llapplyKVLlawinfig(c) then

$3-(3+2)x-1=0$

$x=0.4A$

$V_{th}=V_{AB}=3-3*0.4=1.8V$

<u>Step03</u>-forfindingtheR_{th},allsourcesaresetbezero

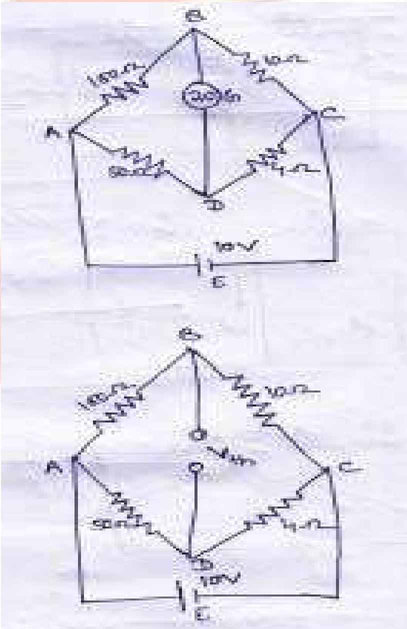
$R_{th}=2//3=(2*3)/(2+3)=1.2Ω$

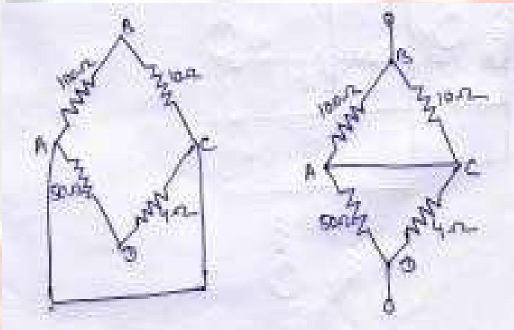
Step04-Thencurrent$I_{th}=1.8/(12.1+1)=0.82A$

Example03: The four arms of a wheatstone bridge have the following resistances .

AB=100Ω,BC=10Ω,CD=4Ω,DA=50Ω.AA galvanometer of 20Ω resistance is connected acrossBD.Usethevenin theorem to compute the current through the galvanometer when the potential difference10V ismaintained across AC.

Solution:





step01-Galvanometerisremoved.

step02-findingtheV_{th}betweenB&D.ABCisapotentialdivideronwhicha voltage drop of 10vtakes place.

PotentialofBw.r.tC=10*10/110=0.909V

Potential of D w.r.t C=10*4/54=.741V

then,

p.dbetweenB&DisV_{th}=0.909-.741=0.168V

Step03-finding R_{th}

removeallsourcestozerokeepingtheirinternalresistances.

$R_{th}=R_{BD}=10//100+50//4=12.79\Omega$
Step04;
lastly$I_{th}=V_{th}/R_{th}+R_L=0.168/(12.79+20)=5mA$

Norton'sTheorem

Statement : In any two terminal active network containing voltage sources and resistances when viewed from its output terminals in equivalent to a constant current source and a parallel resistance. The constant current source is equal to the current which would flow in a short circuit placed across the terminals and parallel resistance is the resistance of the network when viewed from the open circuit side after replacing their internal resistances and removing allthe sources.

OR

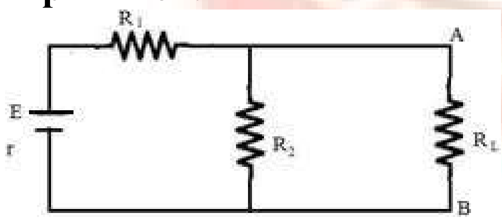
In any two terminal active network the current flowing through the load resistance R_L is given by

$$I_L = \frac{I_{sc}\times R_i}{R_i\times R_L}$$

Where R_i is the internal resistance of the network as viewed from the open ckt side A & B with all sources being replaced by leaving their internal resistancesif any.

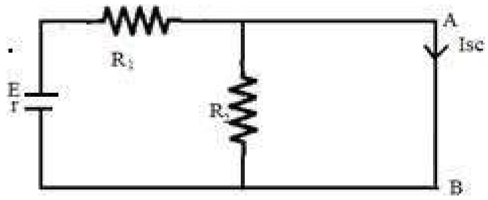
I_{sc}istheshortcktcurrentbetween thetwoterminalsoftheloadresistance when it is shorted

Explanation:



Step–1

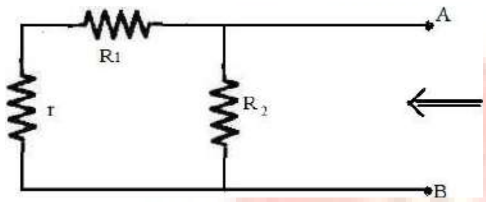
A&BareshortedbyathickcopperwiretofindoutI_{sc}
$I_{sc}=E/(R_1+r)$



I_{sc}=E/ $(R_1$+r)

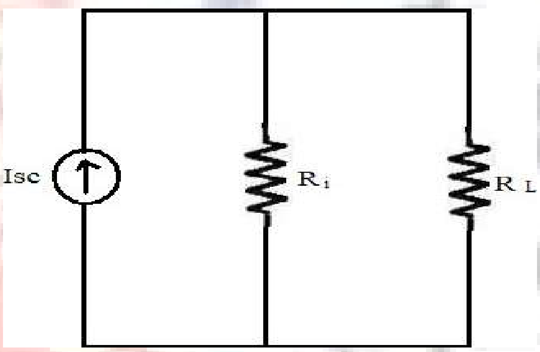
Step–2

Removeallthesourceleavingitsinternalresistanceifanyandviewedfrom opencircuitsideAandBintothenetworktofindR_i.



$R_i=(R_1+r)||R_2$

$R_i=(R_1+r)R_2/(R_1+r+R_2)$



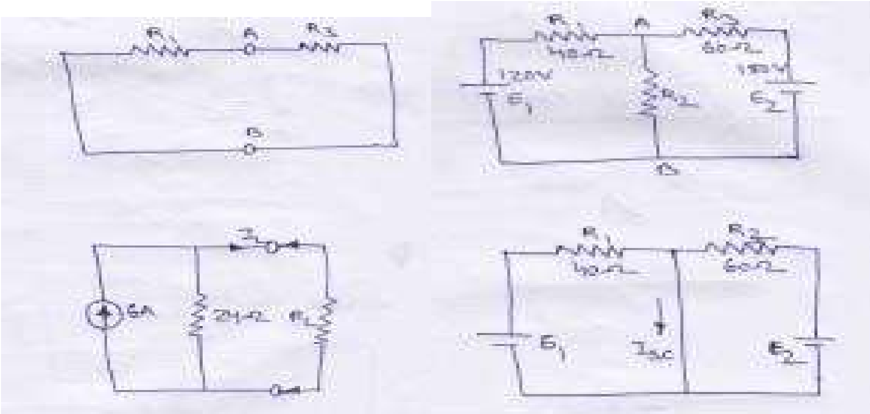
Step–3

ConnectI_{sc}&$R_i$inparallelwithR_L

$$I_L = \frac{I_{sc} \times R_i}{R_i+R_L}$$

Example 01: Usingnorton's theorem find the current that would flow through the resistor R_2whenit takes the values of 12Ω,24Ω&36Ω respectively in the fig shown below.

Solution:



Step 01-remove the load resistance by making short circuit. now terminal ABshort circuited.

Step02-FindingtheshortcircuitcurrentI$_{sc}$

FirstthecurrentduetoE$_1$is=120/40=3A,andduetoE$_2$is180/60=3A. then

I_{sc}=3+3=6A

Step03-findingresistanceR$_N$

Itiscalculatedbybyopencircuittheloadresistanceandviewedfromopen circuit and into the network and all sources are taken zero.

R_N=40//60=(40*60)/(40+60)=24Ω

i) whenR_L=12Ω,I_L=6*24/(24+36)=4A

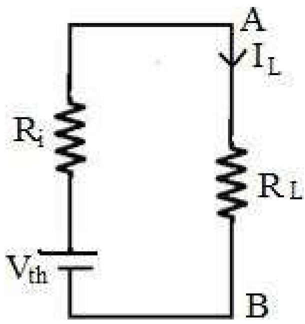
ii) whenR_L=24Ω,I_L=6/2=3A

iii) whenR_L=36Ω,I_L=6*24/(24+36)=2.4A

<u>MaximumPowerTransferTheorem</u>

Statement :A resistive load will abstractmaximum power from a network when the load resistance is equal to the resistance of the network as viewedfrom the output terminals(Open circuit) with all sources removed leaving their internal resistances if any

Proof:



$$I_L = \frac{V_{th}}{R_i + R_L}$$

Powerdeliveredtotheload resistance is given by

$$P = I_L^2 R_L = \left(\frac{V_{th}}{R_i + R_L} \right)^2 R_L$$

$$=\frac{V_{th}^2 R_L}{(R_i+R_L)^2}$$

Power delivered to the load resistance R_L will be maximum

When $\frac{dP_L}{dR_L}=0$

$$\Rightarrow \frac{d}{dR_L}\left\{\frac{V_{th}^2 R_L}{(R_i+R_L)^2}\right\}=0$$

$$\Rightarrow \frac{V_{th}^2(R_i+R_L)^2-V_{th}^2R_L\times 2(R_i+R_L)}{(R_i+R_L)^4}=0$$

$$\Rightarrow V_{th}^2(R_i+R_L)^2-V_{th}^2R_L\times 2(R_i+R_L)=0$$

$$\Rightarrow V_{th}^2(R_i+R_L)^2-2V_{th}^2R_L(R_i+R_L)=0$$

$$\Rightarrow V_{th}^2(R_i+R_L)^2=2V_{th}^2R_L(R_i+R_L)$$

$$\Rightarrow R_i+R_L=2R_L$$

$$\Rightarrow R_i=2R_L-R_L\Rightarrow$$

$$R_i=R_L$$

$$(P_L)\max=\left[\frac{V_{th}^2}{(R_i+R_L)^2}\right]R_L$$

$$=\left(\frac{V_{th}^2}{4R_L^2}R_L\right)$$

$$=\frac{V_{th}^2}{4R_L^2}\times R_L$$

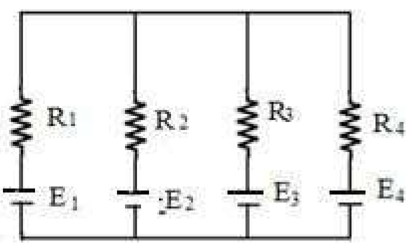
$$\boxed{(P_L)\max=\frac{V_{th}^2}{4R_L}}$$

MILLIMAN'S THEOREM:

According to Millimans Theorem number of sources can be converted into a single source with a internal resistance connected in series to it,if the sources are in parallel connection.

According to the Milliman's theorem the equivalent voltage source

$$E'=\frac{E_1\times\frac{1}{R_1}+E_2\times\frac{1}{R_2}+E_3\times\frac{1}{R_3}+..}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+.....}$$

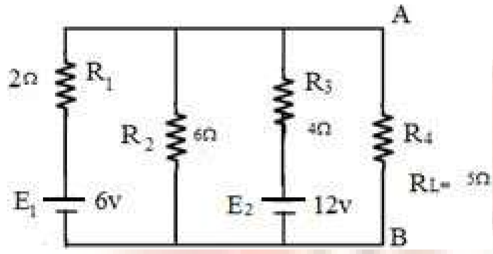


$$=\frac{E_1G_1+E_2G_2+E_3G_3+..}{G_1+G_2+G_3+...}$$

$$=\frac{\frac{E_1}{R_1}+\frac{E_2}{R_2}+\frac{E_3}{R_3}+..}{G_1+G_2+G_3+....}$$

$$=\frac{I_1+I_2+I_3+..}{G_1+G_2+G_3+...}$$

Example – Calculate the current across 5Ω resistor by using Milliman's Thm. Only



Solution:-Given,

$R_1=2\Omega$, $R_2=6\Omega$, $R_3=4\Omega$, $R_L=5\Omega$

$E_1=6v$, $E_2=12v$

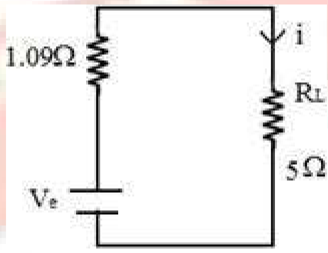
theresistanceR_2isnotcalculatedbecause thereisnovoltagesource

$$V_{oc}=E=\frac{\frac{E_1}{R_1}+\frac{E_2}{R_2}+\frac{E_3}{R_3}}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}}$$

$$=\frac{\frac{6}{2}+0+\frac{12}{4}}{\frac{1}{2}+\frac{1}{6}+\frac{1}{4}}$$

$$=\frac{3+0+3}{\frac{11}{12}}=\frac{6}{11}\times 12=6.54v$$

$$R_1=\frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}}=\frac{1}{\frac{11}{12}}=\frac{12}{11}=1.09$$



$$I_L=\frac{Voc}{1.09+5}=\frac{6.54}{1.09+5}=1.07Amp.$$

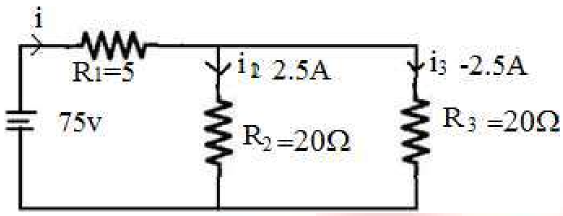
COMPENSATIONTHEOREM:

Statement:

It's states that in a circuit any resistance 'R" in a branch of network in which a current 'I' is flowing can be replaced. For the purposes of calculations by a voltagesource = - IR

OR

If the resistance of any branch of network is changed from R to R +4R where the currentflowing originaly isi. The change current at any other place in the network may be calculated by assuming that one e.m.f – I ΔR has been injected into the modified branch. While all other sources have their e.m.f. suppressed and 'R' represented by their internal resistances only.



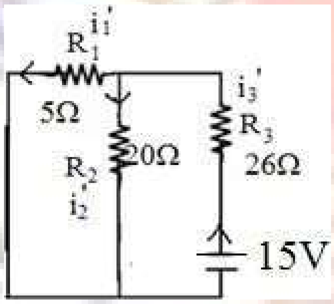
Exp–(01)

Calculatethevaluesofnewcurrentsinthenetworkillustrated ,whenthe resistor R_3 is increased by 30%.

Solution:-Inthegivencircuit,thevaluesofvariousbranchcurrentsare

I_1=75/(5+10)=5A

$I_3 = I_2 = \frac{5 \times 20}{40} = 2.5Amp.$



NowthevalueofR$_3$,whenitincrease30%

R_3=20+(20×0.3)=26Ω

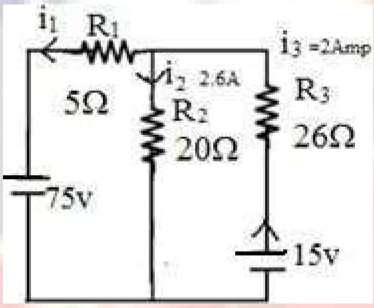
IR=26–20 =6Ω

V=–$I\Delta R$

=–2.5×6

=–15V

$5||20\Omega = \frac{5\times 20}{5+20} = \frac{100}{25} = 4\Omega$



$I'_3 = \frac{15}{4+26} = \frac{15}{30} = 0.5Amp$

$I'_2 = \frac{0.5\times 5}{25} = 0.1Amp$

$I'_1 = \frac{0.5\times 20}{25} = 0.4Amp$

I_1"= 5 – 0.4 = 4.6Amp

I_2"= 0.1+2.5= 2.6Amp

I_3"= 2.5–0.5= 2Amp

RECIPROCITYTHEOREM:

Statement:

It states that in any bilateral network, if a source of e.m.f 'E' in any branch produces a current 'I' any other branch. Then the same e.m.f 'E' acting in the second branch would produce the same current 'I' in the 1st branch.

Step– 1 First ammeter B reads the current in this branch due to the 36v source, the current is given by

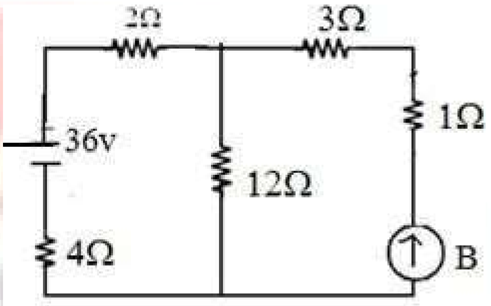
$4||12=\frac{4\times12}{16}=3\Omega$

$R=2+4+3=9\Omega$

$I=\frac{36}{9}=4Amp$

$I_B=\frac{4\times12}{12+3+1}=\frac{48}{16}=3Amp$

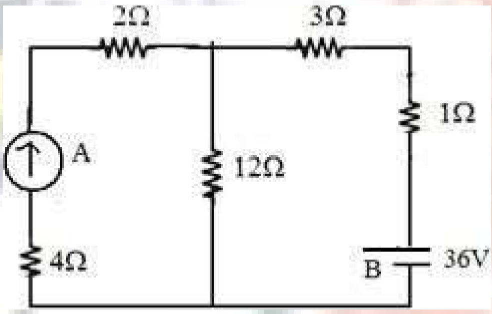
I_B=current through 1Ω resister



Step – (II) Then interchanging the sources and measuring the current

$6\Omega||12\Omega=\frac{6\times12}{6+12}=\frac{72}{18}=4\Omega$

$R=4+3+1=8\Omega$



$I=\frac{36}{8}=4.5Amp$, $I_A=\frac{4.5\times12}{6+2}=3Amp$ Transfer resistance $=\frac{V}{I}=\frac{36}{3}=12\Omega$.

COUPLED CIRCUITS

It is defined as the interconnected loops of an electric network through the magnetic circuit.

There are two types of induced emf.

(1) Statically Induced emf.

(2) Dynamically Induced emf.

Faraday's Laws of Electro-Magnetic :

Introduction→ **First Law:→**

Whenever the magnetic flux linked with a circuit changes, an emf is induced in it.

OR

Wheneveraconductorcutsmagneticfluxanemfisinducedin it.

SecondLaw:→

It states that the magnitude of induced emf is equal to the rate of change of flux linkages.

OR

Theemfinducedisdirectlyproportionaltotherateofchangeoffluxandnumber of turns

Mathematically:

$$e \propto \frac{d\phi}{dt}$$

$$e \propto N$$

Or $$e = -N\frac{d\phi}{dt}$$

Where e=inducedemf

N=No.ofturns ϕ

= flux

'-ve'signisduetoLenz's Law

Inductance:→

Itisdefinedasthepropertyofthesubstancewhichopposesanychangein Current & flux.

Unit:→ Henry

Fleming'sRightHandRule:→

It states that "hold your right hand with fore-finger, middle finger and thumb at right angles to each other. If the fore-finger represents the direction of field, thumbrepresents the directionofmotionofthe conductor, then themiddle finger represents the direction of induced emf."

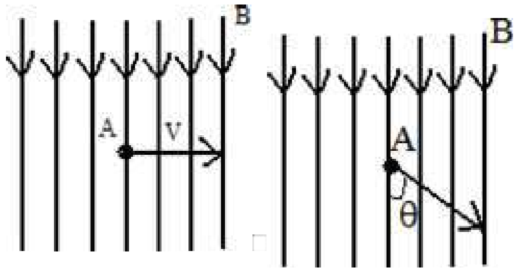
Lenz'sLaw:→

It states that electromagnetically induced current always flows in such a direction that the action of magnetic field set up by it tends to oppose the vary cause which produces it.

OR

Itstatesthatthedirectionoftheinducedcurrent(emf)issuchthatit opposes the change of magnetic flux.

(2) DynamicallyInducedemf:→



In this case the field is stationary and the conductors are rotating in an uniform magnetic field at flux density 'B" Wb/mt^2 and the conductor is lying perpendicular to the magnetic field. Let 'l' is the length of the conductor and it moves a distance of 'dx'nt in time 'dt' second.

Theareasweptbytheconductor=$l.dx$

Hencethefluxcut=$ldx.B$

Changeinfluxintime'dt'second= $\frac{Bldx}{dt}$

$E = \mathbf{Blv}$

Where$V=\frac{dx}{dt}$

Iftheconductorismakinganangle'θ'withthemagneticfield,then

$\boxed{e = Blv\sin\theta}$

(1) StaticallyInducedemf:→

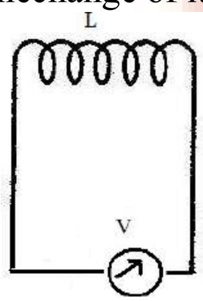
Heretheconductorsareremaininstationaryandfluxlinkedwithit changes by increasing or decreasing.

Itisdividedintotwotypes.

(i) Self-inducedemf.

(ii) Mutually-inducedemf.

(i) Self-induced emf :→It is defined as the emf induced in a coil due to thechange of its ownflux linked with the coil.



Ifcurrentthroughthecoilischangedthenthefluxlinkedwithitsown turn will also change which will produce an emf is called self-induced emf.

Self-Inductance:→

Itisdefinedasthepropertyofthecoilduetowhichitopposesanychange (increase or decrease) of current or flux through it.

Co-efficientofSelf-Inductance(L):→

Itisdefinedastheratioofweberturnsperampereofcurrentinthecoil.

OR

Itistheratiooffluxlinkedperampereofcurrentinthe coil

1stMethodfor'L':→

$$L=\frac{N\phi}{I}$$

Where L=Co-efficientofself-induction N = Number of turns

ϕ= flux

I=Current

2ndMethodforL:→

Weknow that

$$L=\frac{N\phi}{I}$$

$$\Rightarrow LI=N\phi$$

$$\Rightarrow -LI=-N\phi$$

$$\Rightarrow -L\frac{dI}{dt}=-N\frac{d\phi}{dt}$$

$$\Rightarrow -L\frac{dI}{dt}=-N\frac{d\phi}{dt}$$

$$\Rightarrow -L\frac{dI}{dt}=e_L$$

$$\Rightarrow L\frac{dI}{dt}=-e_L$$

$$\boxed{\Rightarrow L=\frac{-e_L}{\frac{dI}{dt}}}$$

WhereL=Inductance

$e_L=-N\frac{d\phi}{dt}$isknownasself-inducedemf.

When $\frac{dI}{dt}=1amp/\sec.$

e=1volt

L=1Henry

Acoilissaidtobea self-inductanceof1Henryif1voltisinducedinit. Whenthecurrentthroughitchangesattherateof1amp/sec.

3rdMethodforL:→

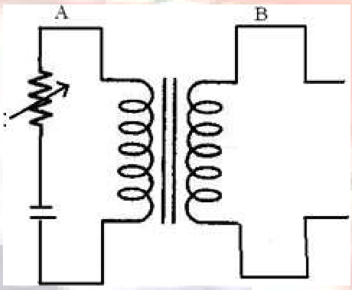
$$L=\frac{M_o M_r A N^2}{l}$$

WhereA=Areaofx-sectionofthecoil N = Number of turns

L=Lengthofthecoil

(ii) MutuallyInducedemf:→

It is defined as the emf induced in one coil due to change in current in other coil. Consider two coils 'A' and 'B' lying close to eachother. An emfwill be induced in coil 'B' due to change of current in coil 'A' by changing the position of the rheostat.



MutualInductance:→

Itisdefinedastheemfinducedincoil'B'duetochangeofcurrentincoil 'A' is the ratio of flux linkage in coil 'B' to 1 amp. Of current in coil 'A'.

Co-efficientofMutualInductance(M)

Coefficient of mutual inductance between the two coils is defined as the weber-turns in one coil due to one ampere current in the other.

1stMethodfor'M':→

$$M=\frac{N_{21}\ \phi}{I_1}$$

N_2 = Number of turns

M=MutualInductance ϕ

$_1$= flux linkage

I_1=Currentin ampere

2ndMethodforM:→

Weknow that

$$M=\frac{N_{21}\phi}{I_1}$$

$$\Rightarrow MI_1=N_2\phi_1$$

$$\Rightarrow -MI_1=N_2\phi_1$$

$$\Rightarrow -M\frac{dI_1}{dt} = -N_2\frac{d\phi}{dt}$$

$$\Rightarrow -M\frac{dI_1}{dt} = e_M$$

$$\Rightarrow M\frac{dI_1}{dt} = -e_M$$

$$\boxed{\Rightarrow M = \frac{-e_M}{\frac{dI_1}{dt}}}$$

Where $e_M = -N_2\frac{d\phi_1}{dt}$ is known as mutually induced emf.

$e_M = -1\,volt$

Then M= 1Henry

A coil is said to be a mutual inductance of 1 Henry when 1 volt is induced when the current of 1 amp/sec. is changed in its neighbouring coil.

3rd Method for M:→

$$M = \frac{M_o M_r A N_1 N_2}{l}$$

Co-efficient of Coupling:

Consider two magnetically coupled coils having N_1 and N_2 turns respectively. Their individual co-efficient of self-inductances are

$$L_1 = \frac{M_o M_r A N_1^2}{l}$$

$$L_2 = \frac{M_o M_r A N_2^2}{l}$$

The flux ϕ_1 produced in coil 'A' due to a current of I_1 ampere is

$$\phi_1 = \frac{L_1 I_1}{N_1} = \frac{M_o M_r A N_1^2}{l} \times \frac{I_1}{N_1}$$

$$\phi_1 = \frac{M_o M_r A N_1 I_1}{l}$$

Suppose a fraction of this flux i.e. $K_1\phi_1$ is linked with coil 'B'

Then $M = \frac{K_1\phi_1}{I_1} \times N_2 = \frac{K_1 N_1 N_2}{l/M_o M_r A}$ ---------------------------------- (1)

Similarly the flux ϕ_2 produced in coil 'B' due to I_2 amp. Is

$$\phi_2 = \frac{M_1 M_r A N_2 I_2}{l}$$

Suppose a fraction of this flux i.e. $K_2\phi_2$ is linked with coil 'A'

Then $M = \frac{K_2\phi_2}{I_2} \times N_1 = \frac{K_2 N_2 N_1}{l/M_o M_r A}$ ---------------------------------- (2)

Multiplying equation (1) & (2)

$$M^2 = \frac{K_1K_2 N_1^2 N_2^2}{\frac{l}{\mu_0\mu_r A}\times\frac{l}{\mu_0\mu_r A}}$$

$$= K^2\left(\frac{\mu_0\mu_r A N_1^2}{l}\right)\left(\frac{\mu_0\mu_r A N_2^2}{l}\right) \qquad [\because K_1 = K_2 = K]$$

$$M^2 = K^2.L_1.L_2$$

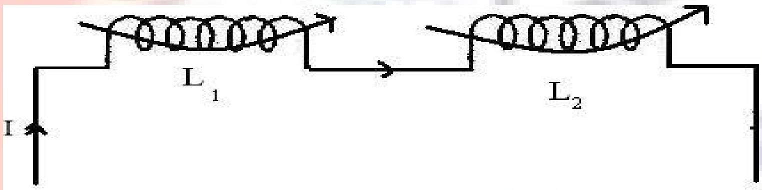
$$K^2 = \frac{M^2.}{L_1.L_2}$$

$$\boxed{\Rightarrow K = \sqrt{\frac{M.}{L_1.L_2}}}$$

Where 'K' is known as the co-efficient of coupling.

Co-efficient of coupling is defined as the ratio of mutual inductance between two coils to the square root of their self- inductances.

Inductances In Series (Additive):→



Fluxes are in the same durection

Let M = Co-efficient of mutual inductance

L_1 = Co-efficient of self-inductance of first coil.

L_2 = Co-efficient of self-inductance of second coil.

EMF induced in first coil due to self-inductance

$$e_{L_1} = -L_1\frac{dI}{dt}$$

Mutually induced emf in first coil

$$e_{M_1} = -M\frac{dI}{dt}$$

EMF induced in second coil due to self induction

$$e_{L_2} = -L_2\frac{dI}{dt}$$

Mutually induced emf in second coil

$$e_{M_2} = -M\frac{dI}{dt}$$

Total induced emf

$$e = e_{L_1} + e_{L_2} + e_{M_1} + e_{M_2}$$

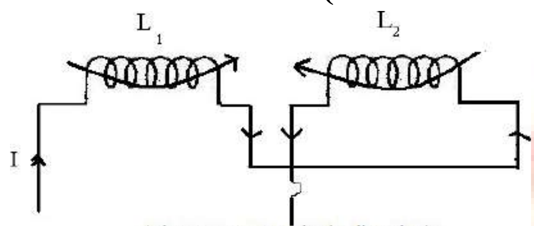
If 'L' is the equivalent inductance, then

$$-L\frac{dI}{dt} = -L_1\frac{dI}{dt} - M\frac{dI}{dt} - L_2\frac{dI}{dt} - M\frac{dI}{dt}$$

$$\Rightarrow -L\frac{dI}{dt} = -\frac{dI}{dt}(L_1 - L_2 - 2M)$$

$$\boxed{\Rightarrow L = L_1 + L_2 + 2M}$$

InductancesInSeries(Substnactive):→



(Fluxes are opposite in direction)

Let M=Co-efficientofmutualinductance

L_1=Co-efficientofself-inductanceoffirstcoil

L_2-=Co-efficientofself-inductanceofsecondcoil Emf induced in first coil due to self induction,

$$e_{L_1} = -L_1\frac{dI}{dt}$$

Mutuallyinducedemfinfirstcoil

$$e_{M_1} = -\left(-M\frac{dI}{dt}\right) = M\frac{dI}{dt}$$

Emfinducedinsecondcoilduetoself-induction

$$e_{L_2} = -L_2\frac{dI}{dt}$$

Mutuallyinducedemfinsecondcoil

$$e_{M_2} = -\left(-M\frac{dI}{dt}\right) = M\frac{dI}{dt}$$

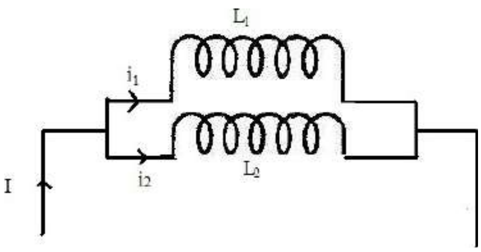
Totalinducedemf

$$e = e_{L_1} + e_{M_1} + e_{L_2} + e_{M_2}$$

Then
$$-L\frac{dI}{dt} = -L_1\frac{dI}{dt} + M\frac{dI}{dt} - L_2\frac{dI}{dt} + M\frac{dI}{dt}$$

$$\Rightarrow -L\frac{dI}{dt} = -\frac{dI}{dt}(L_1 + L_2 - 2M) \qquad \Rightarrow L = L_1 + L_2 - 2M$$

InductancesInParallel:→



LettwoinductancesofL_1&L_2areconnectedinparallel

Lettheco-efficentofmutualinductancebetweenthemisM.

$I=i_1+i_2$

$$\frac{dI}{dt}=\frac{di_1}{dt}+\frac{di_2}{dt} \text{ ---------------------------------- } (1)$$

$$e=L_1\frac{di_1}{dt}+M\frac{di_2}{dt}$$

$$=L_2\frac{di_2}{dt}+M\frac{di_1}{dt}$$

$$\Rightarrow L_1\frac{di_1}{dt}+M\frac{di_2}{dt}=L_2\frac{di_2}{dt}+M\frac{di_1}{dt}$$

$$\Rightarrow (L_1-M)\frac{di_1}{dt}=(L_2-M)\frac{di_2}{dt}$$

$$\Rightarrow \frac{di_1}{dt}=\frac{(L_2-M)di_2}{(L_1-M)dt} \text{ -------------------------------------- } (2)$$

$$\frac{dI}{dt}=\frac{di_1}{dt}+\frac{di_2}{dt}$$

$$=\frac{(L_2-M)di_2}{(L_1-M)dt}+\frac{di_2}{dt}$$

$$\Rightarrow \frac{dI}{dt}=\left(\frac{L_2-M}{L_1-M}+1\right)\frac{di_2}{dt} \text{ -------------------------------- } (3)$$

If'L'istheequivalentinductance

$$e=L\frac{dI}{dt}=L_1\frac{di_1}{dt}+M\frac{di_2}{dt}$$

$$L\frac{dI}{dt}=L_1\frac{di_1}{dt}+M\frac{di_2}{dt}$$

$$\Rightarrow \frac{dI}{dt}=\frac{1}{L}\left(L_1\frac{di_1}{dt}+M\frac{di_2}{dt}\right) \text{ --------------------------- } (4)$$

Substitutingthevalueof$\frac{di_1}{dt}$

$$\frac{dI}{dt}=\frac{1}{L}\left[L_1\frac{L_2-M}{L_1-M}+M\right]\frac{di_2}{dt} \text{ ------------------------------------ } (5)$$

Equatingequation(3)&(5)

$$\left[\left(\frac{L_2-M}{L_1-M}\right)+1\right]\frac{di_2}{dt}=\frac{1}{L}\left[L_1\left(\frac{L_2-M}{L_1-M}\right)+M\right]\frac{di_2}{dt}$$

$$\Rightarrow \frac{L_2-M}{L_1-M}+1=\frac{1}{L}\left[L_1\left(\frac{L_2-M}{L_1-M}\right)+M\right]$$

$$\Rightarrow \frac{L_2-M+L_1-M}{L_1-M}=\frac{1}{L}\left[\frac{L_1L_2-L_1M+L_1M-M^2}{L_1-M}\right]$$

$$\Rightarrow \frac{L_1+L_2-2M}{L_1-M}=\frac{1}{L}\left[\frac{L_1L_2-M^2}{L_1-M}\right]$$

$$\Rightarrow L_1+L_2-2M=\frac{1}{L}\left[L_1L_2-M^2\right]$$

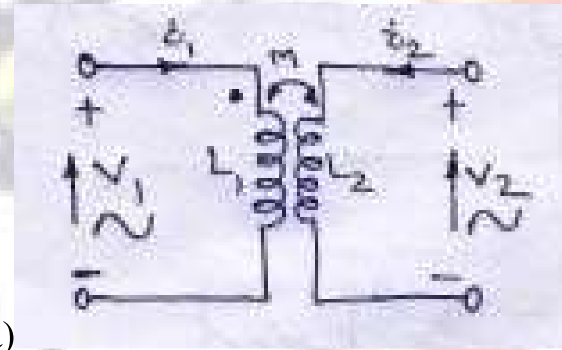
$$\boxed{\Rightarrow L=\frac{L_1L_2-M^2}{L_1+L_2-2M}}$$

When mutual field assist.

$$\boxed{L=\frac{L_1L_2-M^2}{L_1+L_2+2M}}$$

When mutual field opposes.

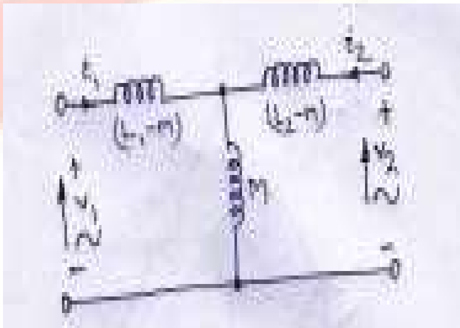
CONDUCTIVELY COUPLED EQUIVALENT CIRCUITS



⇒ The Loop equation are from fig(a)

$$V_1=L_1\frac{di_1}{dt}+M\frac{di_2}{dt}$$

$$V_2=L_2\frac{di_2}{dt}+M\frac{di_1}{dt}$$



⇒ The loop equation are from fig(b)

$$V_1=(L_1-M)\frac{di_1}{dt}+M\frac{d}{dt}(i_1+i_2)$$

$$V_2=(L_2-M)\frac{di_2}{dt}+M\frac{d}{dt}(i_1+i_2)$$

Which,onsimplificationbecome

$$V_1=L_1\frac{di_1}{dt}+M\frac{di_2}{dt}$$

$$V_2=L_2\frac{di_2}{dt}+M\frac{di_1}{dt}$$

Socalledconductivelyequivalentofthemagneticcircuit. Herewemay representZ$_A$ = L$_1$-M .

Z$_B$= (L$_2$-M)andZ$_C$=M

In case M is + ve and both the currents then $Z_A = L_1$-M , $Z_B = L_2$-Mand Z_C =M, also , ifis – ve and currents in the common branch opposite to each other Z_A = L_1+M , Z_B = L_2+Mand Z_C = - M.

Similarly, if M is –ve but the two currents in the common branch are additive,then also.

$Z_A=L_1+M, Z_B=L_2+M$and$Z_C=-M$.

FurtherZ_A , Z_B and Z_C may also be assumed to be the T equivalent of the circuit.

Exp.-01:

Two coupled cols have self inductances$L_1= 10\times10^{-3}$H and $L_2= 20\times10^{-3}$H. The coefficient of coupling (K) being 0.75 in the air, find voltage in the second coil and the flux of first coil provided the second coils has 500 turns and the circuit current is given by i_1 = 2sin 314.1A.

Solution:

$$M=K\sqrt{L_1L_2}$$

$$M=0.75\sqrt{10\times10^{-3}\times20\times10^{-3}}$$

$$\Rightarrow M=10.6\times10^{-3}H$$

Thevoltageinducedinsecondcoil is

$$v_2=M\frac{di_1}{dt}=M\frac{di}{dt}$$

$$=10.6\times10^{-3}\frac{d}{dt}(2\sin314t)$$

$$=10.6\times10^{-3}\times2\times314\cos314t.$$

ThemagneticCKtbeinglinear,

$$M=\frac{N_2\phi_{2}}{i_1}=\frac{500\times(K\phi)}{i_1}$$

$$\phi=\frac{M}{500\times K}\times i_1=\frac{10.6\times10^{-3}}{500\times0.75}\times2\sin314t$$

$= 5.66\times10^{-5}\sin314t$

$\phi_1=5.66\times10^{-5}\sin s314t$.

Exp.02

Find the total inductance of the three series connected coupled coils.Where the self and mutual inductances are

$L_1 = 1H$, $L_2 = 2H$, $L_3 = 5H$

$M_{12}=0.5H$,$M_{23}=1H$,$M_{13}=1H$

Solution:

$L_A = L_1+M_{12}+M_{13}$
$=1+20.5+1$
$=2.5H$

$L_B = L_2+M_{23}+M_{12}$
$=2+1+0.5$
$=3.5H$

$L_C = L_3+M_{23}+M_{13}$
$=5+1+1$
$=7H$

Total inductances are

$L_{ea} = L_A+L_B+L_c$
$=2.5+3.5+7$
$=13H$(Ans)

Example03:

Two identical 750 turn coils A and B lie in parallel planes. A current changing at the rate of 1500A/s in A induces an emf of 11.25 V in B. Calculate the mutual inductance of the arrangement .If the self inductance of each coil is 15mH, calculate the flux produced in coil A per ampere and the percentage of this flux which links the turns of B.

Solution: Weknowthat

$$e_M = \frac{MdI_1}{dt}$$

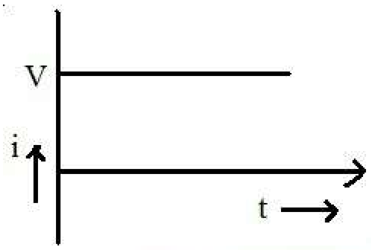
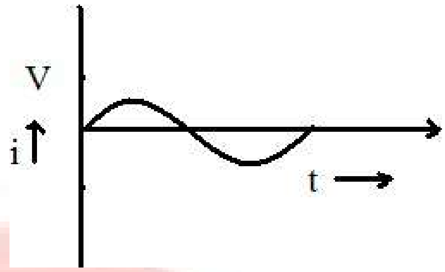
$$M = {e_M}\Big/{\frac{dI_1}{dt}} = \frac{11.25}{1500} = 7.5mH$$

now,

$$L_1 = \frac{N_1\varphi_1}{I_1} = \frac{\varphi_1}{I_1} = \frac{L_1}{N_1} = 15 * \frac{10^{-3}}{750} = 2 * 10^{-5}\text{Wb/A}$$

$$k = \frac{M}{\sqrt{L_1L_2}} = \frac{7.5 * 10^{-3}}{15 * 10^{-3}} = 0.5 = 50\%$$

A.CFUNDAMENTAL

	DirectCurrent		**AlternatingCurrent**
			
(1)	D.C. always flow in onedirection and whose magnitude remains constant.	(1)	A.C. is one which reverse periodically in direction and whose magnitude undergoes a definite cycle changes in definite intervals of time.
(2)	Highcostofproduction.	(2)	Lowcostofproduction
(3)	It is not possible by D.C.Because D.C.isdangerous to the transformer.	(3)	ByusingtransformerA.C.voltage can be decreased or increased.
(4)	Itstransmissioncostistoohigh.	(4)	A.C.canbetransmittedtoalong distance economically.

DefinitionofA.C.terms:-

Cycle: Itisonecompletesetof+veand–vevaluesofalternatingquality spread over 360°or 2]]radan.

TimePeriod: Itisdefinedasthetimerequiredtocompleteonecycle.

Frequency: Itisdefinedasthereciprocaloftimeperiod.i.e. $f=1/T$

Or

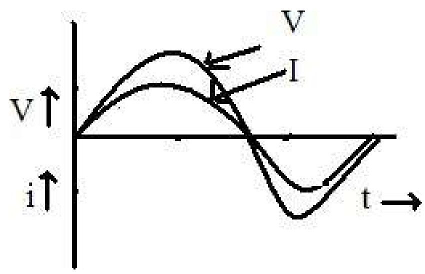
Itisdefinedasthenumberofcyclescompletedpersecond.

Amplitude : It is defined as the maximum value of either +ve half cycle or –ve half cycle.

Phase: Itisdefinedastheangulardisplacementbetweentwohavesiszero.

OR

Two alternating quantity are inphase when each pass through their zero value at the same instant and also attain their maximum value at the same instant in a given cycle.



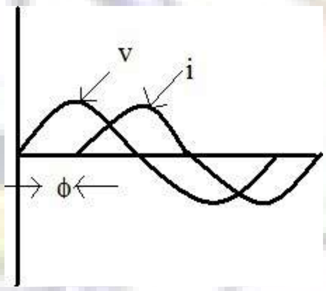
$V=V_m sinwti$

$=I_m \sin wt$

PhaseDifference:-Itisdefinedastheangulardisplacementbetweentwo alternating quantities.

OR

If the angular displacement between two waves are not zero, then that is known as phase difference. i.e. at a particular time they attain unequal distance.



OR

Two quantities are out of phase if they reach their maximum value or minimumvalueatdifferenttimesbutalwayshaveanequalphaseanglebetween them.

Here $V=V_m sinwt$

$i=I_m sin(wt-\phi)$

Inthiscasecurrentlagsvoltagebyanangle‘ϕ’.

PhasorDiagram:

GenerationofAlternatingemf:-

Consider a rectangular coil of ‘N” turns, area of cross-section is ‘A’ nt^2 is placed in x-axis in an uniform magnetic field of maximum flux density *Bm web/nt²*. The coil is rotating in the magnetic field with a velocity of w radian / second. Attime t = 0, the coil is in x-axis. After interval of time ‘dt’ second the coil make rotating in anti-clockwise direction and makes an angle ‘θ’ with x-direction.The perpendicular component of the magnetic field is $\phi= \phi n \cos wt$

AccordingtoFaraday’sLawsofelectro-magneticInduction

$$e = -N\frac{d\phi}{dt}$$

$$= -N\left(\frac{d}{dt}\phi_m \cos wt\right)$$

$$= -N(-\phi_m w \cos wt)$$

$$= N w \phi_m \sin wt$$

$$= 2\pi f N \phi_m \sin wt \quad (\because w = 2\pi f)$$

$$= 2\pi f N B_m A \sin wt$$

$$e = E_m \sin wt$$

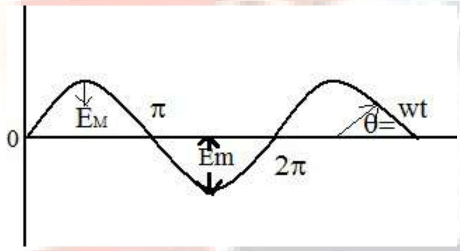
Where $E_m = 2\pi f N B_m A$

f→frequencyin Hz

B_m→MaximumfluxdensityinWb/mt^2

Nowwhen θorwt=90°e = E_m

i.e. $E_m = 2\pi f N B_m A$



RootMeanSquare(R.M.S)Value:→

The r.m.s. value of an a.c. is defined by that steady (d.c.) current which when flowing through a given circuit for a given time produces same heat as produced by the alternating current when flowing through the same circuit for the same time.

Sinuscdialalternatingcurrentis $i = I_m \sin wt = I_m \sin\theta$

Themeanofsquaresoftheinstantaneousvaluesofcurrentoverone completecycle

$$= \int_0^{2\pi} \frac{i^2 . d\theta}{(2\pi - 0)}$$

Thesquarerootofthisvalueis

$$= \sqrt{\int_0^{2\pi} \frac{i^2 . d\theta}{2\pi}}$$

$$= \sqrt{\int_0^{2\pi} \frac{(I_m \sin\theta)^2}{2\pi} d\theta}$$

$$= \sqrt{\frac{I_m^2}{2\pi} \int_0^{\pi} \sin^2\theta \, d\theta}$$

$$= \sqrt{\frac{I_m^2}{2\pi} \int_0^{\pi} \left(\frac{1-\cos 2\theta}{2}\right) d\theta}$$

$$= \sqrt{\frac{I_m^2}{4\pi} \int_0^{\pi} (1-\cos 2\theta) \, d\theta}$$

$$= \sqrt{\frac{I_m^2}{4\pi} \left[\theta - \frac{\sin 2\theta}{2}\right]_0^{2\pi}}$$

$$= \sqrt{\frac{I_m^2}{4\pi} \int_0^{\pi} \left(2\pi - \frac{\sin 4\pi}{2}\right) d\theta}$$

$$= \sqrt{\frac{I_m^2}{4\pi} \int_0^{\pi} (2\pi - 0)}$$

$$= \sqrt{\frac{I_m^2}{2}} = \frac{I_m}{\sqrt{2}}$$

$$I_{r.m.s} = \frac{I_m}{\sqrt{2}} = 0.707 I_m$$

AverageValue:→

The average value of an alternating current is expressed by that steady current (d.c.) which transfers across any circuit the same charge as it transferred by that alternating current during the sae time.

The equation ofthealternating currentisi= $I_m \sin\theta$

$$I_{av} = \int_0^{\pi} \frac{i \, d\theta}{(\pi - 0)}$$

$$= \int_0^{\pi} \frac{I_m \sin\theta}{\pi} d\theta = \frac{I_m}{\pi} \int_0^{\pi} \sin\theta \, d\theta$$

$$= \frac{I_m}{\pi} \left[-\cos\theta\right]_0^{\pi} = \frac{I_m}{\pi} \left[-\cos\pi - (\cos 0^0)\right]$$

$$= \frac{I_m}{\pi} \left[1 - 0(-1)\right]$$

$$I_{av} = \frac{2 I_m}{\pi}$$

$$I_{av} = \frac{2 \times Maximum Current}{\pi}$$

Hence, $I_{av} = 0.637 I_m$

Theaveragevalueoveracompletecycleiszero

Amplitude factor/ Peak factor/ Crest factor :- It is defined as the ratio of maximum value to r.m.s value.

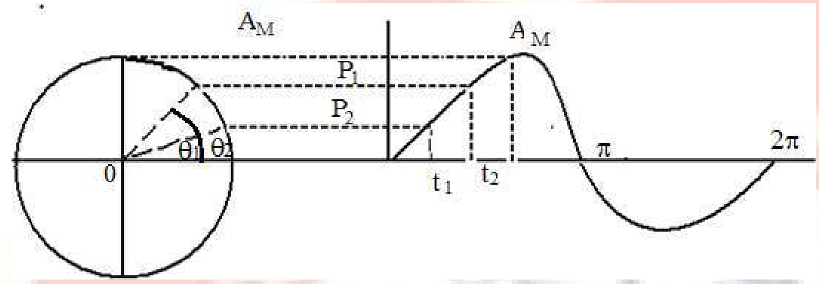
$$Ka=\frac{Maximum\,Value}{R.M.S.Value}=\frac{I_m}{\frac{I_m}{\sqrt{2}}}=\sqrt{2}=1.414$$

Formfactor:-Itisdefinedas theratioofr.m.svaluetoaveragevalue.

$$Kf=\frac{r.m.s.Value}{Average.Value}=\frac{0.707I_m}{0.637I_m}=\sqrt{2}=1.414$$

$$Kf=1.11$$

PhasororVectorRepresentationofAlternatingQuantity:→



An alternating current or voltage, (quantity) in a vector quantity whichhas magnitude as well as direction. Let the alternating value of current be represented by theequation e = E_m Sin wt. The projection of E_m on Y-axis at any instant gives the instantaneous value of alternating current. Since the instantaneous values are continuously changing, so they are represented by a rotating vector or phasor. A phasor is a vector rotating at a constant angular velocity

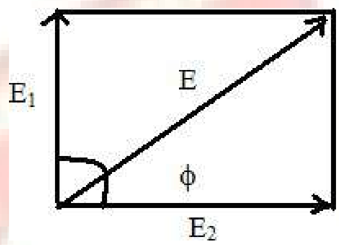
Att_1,$e_1=E_m\sin wt_1$

Att_2,$e_2=E_m\sin wt_2$

AdditionoftwoalternatingCurrent:→

Let$e_1=E_m\sin wt$

$e_2=E_m\sin(wt-\phi)$



The sum of two sine waves of thesame frequency is another sine wave of samefrequency but of a different maximum value and Phase.

$$e=\sqrt{e_1^2+e_2^2+2e_1e_2\cos\phi}$$

PhasorAlgebra:→

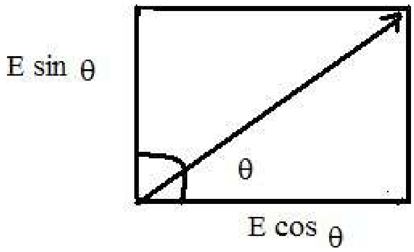
Avectorquantitycanbeexpressedintermsof

(i) RectangularorCartesianform

(ii) Trigonometricform

(iii) Exponentialform

(iv) Polarform



$E = a + jb$

$= E(\cos\theta + j\sin\theta)$

Where a = E cos θ is the active part

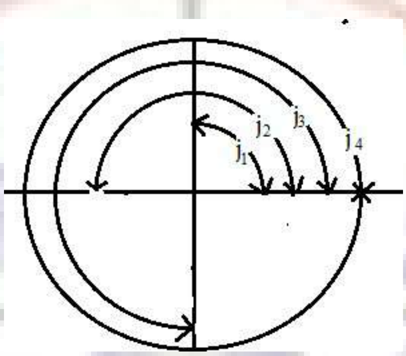
b = E sin θ is the reactive part

$\theta = \tan^{-1}\left(\frac{b}{a}\right)$ = Phase angle

$j = \sqrt{-1}\,(90^o)$ $j^2 = -1(180^o)$

$j^3 = -j(270^o)$

$j^4 = 1(360^o)$



(i) Rectangularfor:-

$E = a \pm jb$

$\tan\theta = b/a$

(ii) Trigonometricform:-

$E = E(\cos\theta \pm j\sin\theta)$

(iii) Exponentialform:-

$E = Ee^{\pm j\theta}$

(iv) Polarform:-

$E = E/\pm e$ $\quad (E = \sqrt{a^2+b^2}\ )$

AdditionorSubtration:-

$E_1 = a_1 + jb_1$ $E_2 = a_2 + jb_2$

$E_1 \pm E_2 = (a_1 + a_2) \pm (b_1 + b_2)$

$\phi = \tan^{-1}\left(\frac{b_1 + b_2}{a_1 + a_2}\right)$

Multiplication:-

$E_1 \times E_2 = (a_1 + ja_1) \pm (a_1 + jb_2)$

$= (a_1a_2 - b_1b_2) + j(a_1a_2 + b_1b_2)$

$$\phi = \tan^{-1}\left(\frac{a_1b_2+b_1a_2}{a_1a_2-b_1b_2}\right)$$

$E_1=E_1\angle\theta_1$

$E_2=E_2\angle\theta_2$

$E_1\times E_2=E_1E_2\angle\phi_1+\phi_2$

Division:-

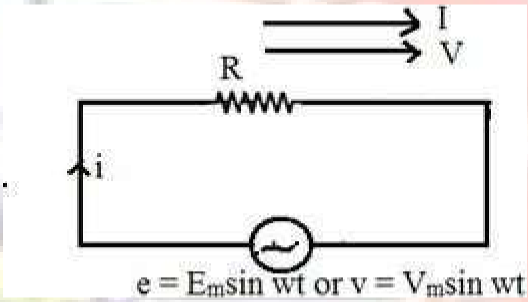
$E_1=E_1\angle\theta_1$

$E_2=E_2\angle\theta_2$

$$\frac{E_1}{E_2}=\frac{E_1\angle\theta_1}{E_2\angle\theta_2}=\frac{E_1}{E_2}\angle\theta_1-\theta_2$$

A.C.throughPureResistance:→

LettheresistanceofRohmisconnectedacrosstoA.Csupplyofapplied voltage



$e=E_m\sin wt$ -------------------- (1)

Let'I'istheinstantaneouscurrent.

Here $e=iR$

$\Rightarrow i = e/R$

$i=E_m\sin wt/R$ ------------------ (2)

Bycomparingequation(1)andequation(2)wegetalternatingvoltage and current in a pure resistive circuit are in phase

Instantaneouspowerisgivenby P

$= ei$

$=E_m\sin wt.I_m\sin wt$

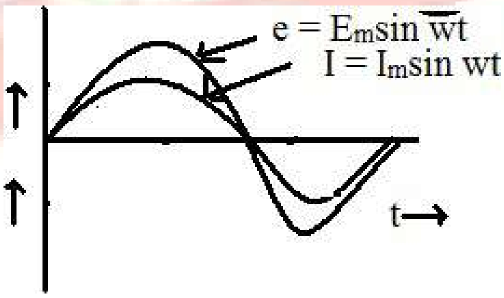
$=E_mI_m\sin^2 wt$

$=\frac{E_mI_m}{2}.2\sin^2 wt$

$=\frac{E_m}{\sqrt{2}}.\frac{I_m}{\sqrt{2}}.(1-\cos 2wt)$

$P=\frac{E_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}-\frac{E_m}{\sqrt{2}}.\frac{I_m}{\sqrt{2}}.\cos 2wt$

i.e. $P=\frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}-\frac{V_m}{\sqrt{2}}.\frac{I_m}{\sqrt{2}}.\cos 2wt$



Where $\frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}$ is called constant part of power.

$\frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}.\cos 2wt$ is called fluctuating part of power.

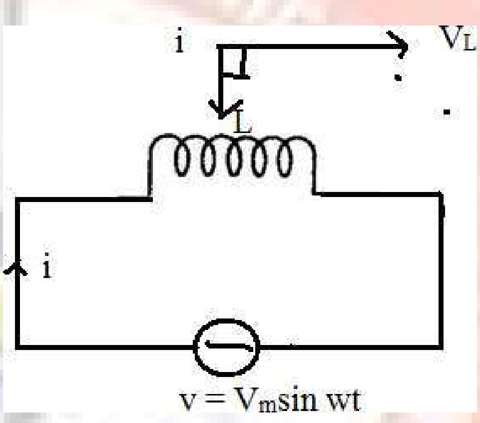
The fluctuating part $\frac{V_m I_m}{2}.\cos 2wt$ of frequency double that of voltage and current waves.

Hence power for the whole cycle is $P=\frac{V_m}{\sqrt{2}}.\frac{I_m}{\sqrt{2}}=V_{rms}.I_{rms}$

$\boxed{\Rightarrow P=VI \text{ watts}}$

A.C through Pure Inductance:→

Let inductance of 'L' henry is connected across the A.C. supply

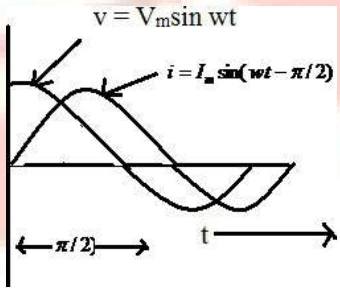


$v=V_m\sin wt$ --------------------(1)

According to Faraday's laws of electromagnetic inductance the emf induced across the inductance

$$V=L\frac{di}{dt}$$

$\frac{di}{dt}$ is the rate of change of current



$$V_m\sin wt=L\frac{di}{dt}$$

$$di=\frac{V_m\sin wt\,dt}{L}$$

$$\Rightarrow di=\frac{V_m}{L}\sin wt.dt$$

Integrating both sides,

$$\int di=\frac{V_m}{L}\int \sin wt.dt$$

$$i=\frac{V_m}{L}\left(-\frac{\cos wt}{w}\right)$$

$$i = -\frac{V_m \cos wt}{wL}$$

$$i = -\frac{V_m}{wL}\cos wt$$

$$i = -\frac{V_m}{wL}\sin\left(wt - \frac{\pi}{2}\right)$$

$$= -\frac{V_m}{X_L}\sin\left(wt - \frac{\pi}{2}\right) \qquad [\because X_L = 2\pi fL = wL]$$

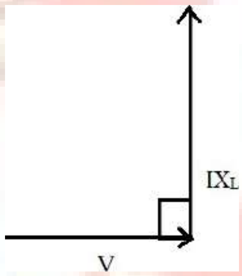
Maximum value of i is $I_m = \frac{V_m}{X_L}$ when $\sin\left(wt - \frac{\pi}{2}\right)$ is unity.

Hence the equation of current becomes $i = I_m \sin(wt - \pi/2)$

So we find that if applied voltage is rep[resented by $v = V_m \sin wt$, then current flowing in a purely inductive circuit is given by

$$i = I_m \sin(wt - \pi/2)$$

Here current lags voltage by an angle $\pi/2$ Radian.



Power factor $= \cos\phi$

$= \cos 90^\circ$

$= 0$

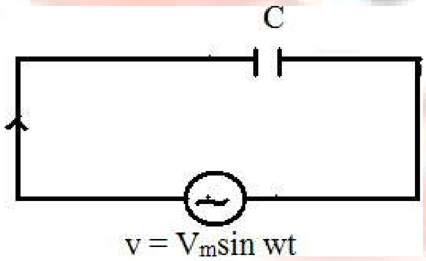
Power Consumed $= VI\cos\phi$

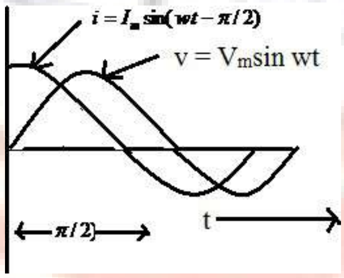
$= VI \times 0$

$= 0$

Hence, the power consumed by a purely Inductive circuit is zero.

A.C. Through Pure Capacitance:→





Let a capacitance of ‘C” farad is connected across the A.C. supply of applied voltage

$v = V_m \sin wt$ --------------------------(1)

Let ‘q’ = change on plates when p.d. between two plates of capacitor is ‘v’

$q = cv$

$q = cV_m \sin wt$

$$\frac{dq}{dt}=c\frac{d}{dt}(V_m\sin wt)$$

$$i=cV_m sinwt$$

$$=wcV_m coswt$$

$$=\frac{V_m}{1/wc}\cos wt$$

$$=\frac{V_m}{Xc}\cos wt \qquad \left[\because X_c=\frac{1}{wc}=\frac{1}{2\pi fc}\right.$$ isknownascapacitivereactance in ohm.]

$$=I_m\cos wt$$

$$=I_m\sin(wt+\pi/2)$$

Herecurrentleadsthesupplyvoltagebyanangle $\pi/2$ radian.

Powerfactor $= \cos\phi$

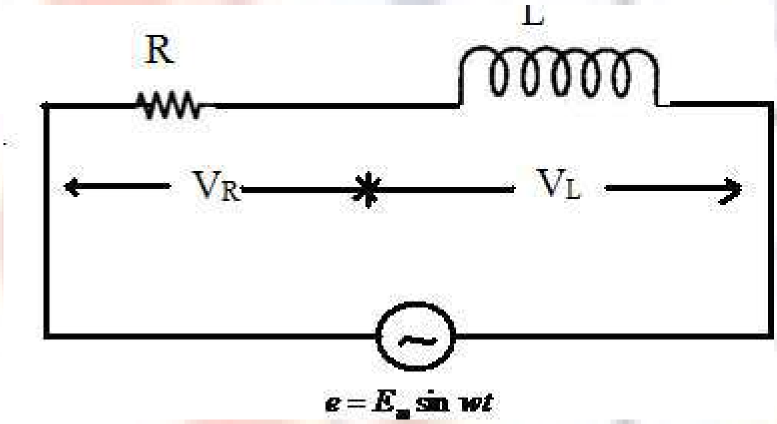
$= \cos 90^\circ = 0$

Power Consumed $= VI\cos\phi$

$= VI\times 0 = 0$

Thepowerconsumedbyapurecapacitivecircuitiszero.

A.C.ThroughR-LSeriesCircuit:→



TheresistanceofR-ohmandinductanceofL-henryareconnectedinseries across the A.C. supply of applied voltage

$e=E_m\sin wt$ --------------------------------(1)

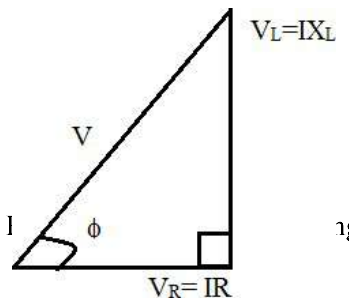
$$V=V_R+jV_L$$

$$=\sqrt{V_R^2+V_L^2}\angle\phi=\tan^{-1}\left(\frac{X_L}{R}\right)$$

$$=\sqrt{(IR)^2+(IX_L)^2}\angle\phi=\tan^{-1}\left(\frac{X_L}{R}\right)$$

$$=I\sqrt{R^2+X_L^2}\angle\phi=\tan^{-1}\left(\frac{X_L}{R}\right)$$

$$V=IZ\angle\phi=\tan^{-1}\left(\frac{X_L}{R}\right)$$



Where $Z = \sqrt{R^2 + X_L^2}$

$= R + jX_L$ is known as impedance of R-L series Circuit.

$$I = \frac{V}{Z\angle\phi} = \frac{E_m \sin wt}{Z\angle\phi}$$

$I = I_m \sin(wt - \phi)$

Here current lags the supply voltage by an angle ϕ.

Power Factor:→ It is the cosine of the angle between the voltage and current.

OR

It is the ratio of active power to apparent power.

OR

It is the ratio of resistance to inpedence.

Power:→

$= v.i$

$= V_m \sin wt . I_m \sin(wt - \phi)$

$= V_m I_m \sin wt . \sin(wt - \phi)$

$= \frac{1}{2} V_m I_m \, 2 \sin wt . \sin(wt - \phi)$

$= \frac{1}{2} V_m I_m [\cos\phi - \cos(2wt - \phi)]$

Obviously the power consists of two parts.

(i) a constant part $\frac{1}{2} V_m I_m \cos\phi$ which contributes to real power.

(ii) a pulsating component $\frac{1}{2} V_m I_m \cos(2wt - \phi)$ which has a frequency twice that of the voltage and current. It does not contribute to actual power since its average value over a complete cycle is zero.

Hence average power consumed

$$= \frac{1}{2} V_m I_m \cos\phi$$

$$= \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\phi$$

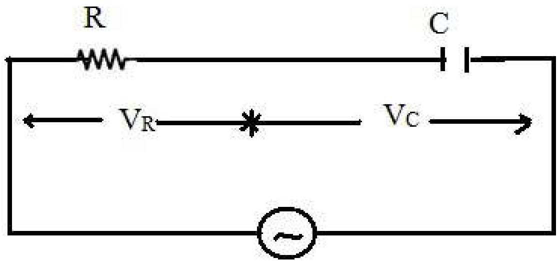
$$= VI \cos\phi$$

Where V & I represents the r.m.s value.

A.C. Through R-C Series Circuit:→

The resistance of 'R'-ohm and capacitance of 'C' farad is connected across the A.C. supply of applied voltage

$e = E_m \sin wt$ ---(1)



$V = V_R + (-jV_C)$

$= IR + (-jIX_C)$

$= I(R - jX_C)$

$V = IZ$

Where $Z = R - jX_C = \sqrt{R^2 + X_C^2}$ isknownasimpedanceofR-Cseries Circuit.

$Z = R - jX_C$

$= \sqrt{R^2 + X_C^2}$

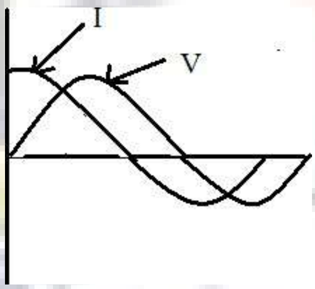
$\angle -\phi = \tan^{-1}\left(\frac{X_C}{R}\right)$

$V = IZ\angle -\phi$

$\Rightarrow I = \frac{V}{Z\angle -\phi}$

$= \frac{E_m \sin wt}{Z\angle -\phi}$

$= \frac{E_m}{Z\angle}\sin(wt + \phi)$

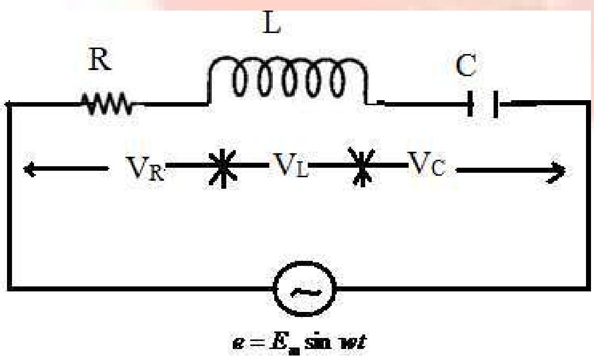


$\Rightarrow I = I_m \sin(wt + \phi)$

Herecurrentleadsthesupplyvoltagebyanangle'ϕ'.

A.C.ThroughR-L-CSeriesCircuit:→

Letaresistanceof'R'-ohminductanceof'L'henryandacapacitanceof'C' faradareconnectedacrosstheA.C.supplyinseriesofappliedvoltage



$e = E_m \sin wt$ ---------------------------(1)

$$e = \vec{V_R} + \vec{V_L} + \vec{V_C}$$
$$= V_R + jV_L - jV_C$$
$$= V_R + j(V_L - V_C)$$
$$= I_R + j(IX_L - IX_C)$$
$$= I[R + j(X_L - X_C)]$$
$$= I\sqrt{R^2 + (X_L - X_C)^2} \angle \pm\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$
$$= IZ\angle \pm\phi$$

Where $Z = I\sqrt{R^2 + (X_L - X_C)^2}$ isknownastheimpedanceofR-L-CSeries Circuit.

If$X_L > X_C$,thentheangleis+ve. If$X_L <$ X_C,then the angleis-ve.

Impedanceisdefinedasthephasorsumofresistanceandnetreactance

$$e = IZ\angle \pm\phi$$
$$\rightarrow I = \frac{e}{Z\angle \pm\phi} IZ\angle \pm\phi = \frac{E_m \sin wt}{Z\angle \pm\phi} = I_m \sin(wt \pm\phi)$$

(1) If $X_L > X_C$,then P.f willbe lagging.

(2) If $X_L < X_C$,then,P.fwillbe leading.

(3) If $X_L = X_C$,then,thecircuitwillberesistiveone.Thep.f.becomesunity andtheresonanceoccurs.

REASONANCE

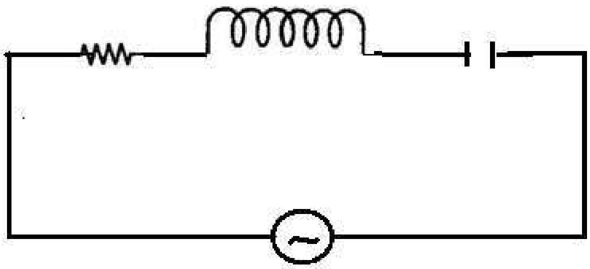
_It is defined as the resonance in electrical circuit having passive or active elements represents a particular state when the current and the voltage in the circuitismaximumandminimumwithrespecttothemagnitudeofexcitationat a particular frequency and the impedances being either minimum or maximum at unity power factor

Resonanceareclassifiedintotwotypes.

(1) SeriesResonance

(2) ParallelResonance

(1) SeriesResonance:- Letaresistanceof'R'ohm,inductanceof'L' henryandcapacitanceof'C'faradareconnectedinseriesacrossA.C.supply



$e = E_m \sin wt$

Theimpedanceofthecircuit

$Z = R + j(X_L - X_C)]$

$Z = \sqrt{R^2 + (X_L - X_C)^2}$

Theconditionofseriesresonance:

Theresonancewilloccurwhenthereactivepartofthelinecurrentiszero Thep.f. becomes unity.

The net reactance will be zero.

The current becomes maximum.

Atresonancenetreactanceiszero

$X_L - X_C = 0$

$\Rightarrow X_L = X_C$

$\Rightarrow W_o L = \frac{1}{W_o C}$

$\Rightarrow W_o^2 LC = 1$

$\Rightarrow W_o^2 = \frac{1}{LC}$

$\Rightarrow W_o = \frac{1}{\sqrt{LC}}$

$\Rightarrow 2\pi f_o = \frac{1}{\sqrt{LC}}$

$\Rightarrow f_o = \frac{1}{2\pi\sqrt{LC}}$

Resonantfrequency$(f_o) = \frac{1}{2\pi} \cdot \frac{1}{\sqrt{LC}}$

ImpedanceatResonance

$Z_0 = R$

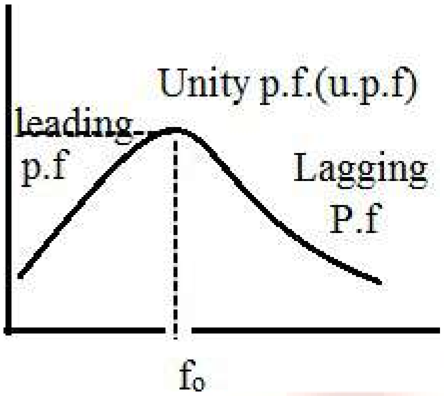
CurrentatResonance

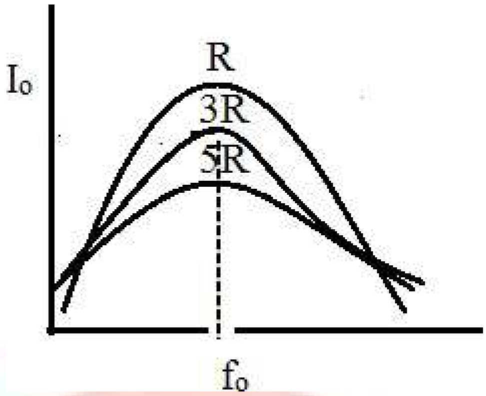
$I_o = \frac{V}{R}$

Powerfactoratresonance

$p.f. = \frac{R}{Z_o} = \frac{R}{R} = 1 \qquad [\because Z_o = R]$

ResonanceCurve:-





At low frequency the X_c isgreater and the circuit behavesleading and at high frequency the X_L becomes high and the circuitbehaves lagging circuit.

Iftheresistancewillbelowthecurvewillbestiff(peak).

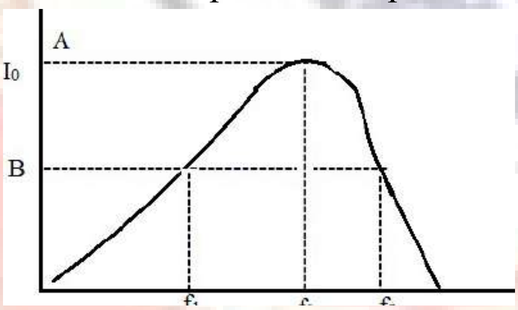
- If the resistance will go oh increasing the current goes on decreasing and the curve become flat.

BandWidth:→

At point 'A'the power lossis $I_0{}^2R$. Thefrequencyisf$_0$whichisatresonance.

Atpoint'B'thepowerlossis $\frac{I_0^2R}{2}$.

Thepowerlossis50%ofthepowerlossatpoint 'A"/



Hencethefrequencies correspondingtopoint'B'isknownashalfpowerfrequenciesf_1&f_2.

f_1=Lowerhalfpowerfrequency

$$f_1=f_0-\frac{R}{4\pi L}$$

F_2=Upperhalfpower frequency

$$f_2=f_0+\frac{R}{4\pi L}$$

Band width(B.W.)isdefinedasthedifferencebetweenupperhalfpowerfrequency ad lower half power frequency.

$$B.W.=f_2-f_1=\frac{R}{2\pi L}$$

Selectivity:→

Selectivityisdefinedastheratioof Bandwidthtoresonantfrequency

$$\text{Selectivity}=\frac{B.W.}{f_0}=\frac{R}{2\pi L} \qquad Selectivity=\frac{R}{2\pi f_o L}$$

QualityFactor(Q-factor):→

It isdefined asthe ratio of2π×Maximum energy storedto energy dissipated per cycle

$$\text{Q-factor} = \frac{2\pi\times\frac{1}{2}LI_0^2}{I^2RT}$$

$$=\frac{\pi L\left(\sqrt{2}I\right)^2}{I^2RT}$$

$$=\frac{\pi L.2I^2}{I^2RT}$$

$$=\frac{\pi L.2I^2}{I^2RT}$$

$$=\frac{2L\pi}{RT}$$

$$\boxed{\text{Qualityfactor}=\frac{2f_0L.\pi}{R}} \qquad \left[Q=\frac{1}{T}=f_0\right]$$

Qualityfactorisdefinedasthereciprocalofpowerfactor.

$$\boxed{\text{Qfactor}=\frac{1}{\cos\phi}}$$

Itisthereciprocalofselectivity.

$$\text{Q-factorOrMagnificationfactor}=\frac{\text{VoltageacrossInductor.}}{\text{Voltage across resistor}}$$

$$=\frac{I_0X_L}{I_0R}$$

$$=\frac{X_L}{R}$$

$$=\frac{2f_0L}{R}=\frac{W_0L}{R}$$

$$\boxed{\text{Q-factor}=\frac{W_0L}{R}}$$

$$\text{Q-factorfactor}=\frac{\text{VoltageacrossCapacotor.}}{\text{Voltageacrossresistor}}$$

$$=\frac{I_0X_c}{I_0R}$$

$$=\frac{X_C}{R}$$
$$=\frac{1}{2\pi f_0 C}=\frac{1}{2\pi f_0 CR}$$

$$\boxed{\text{Q-factor}=\frac{1}{W_0 CR}}$$

$$Q^2=\frac{W_0L}{R}\times\frac{1}{W_0CR}$$
$$Q^2=\frac{1}{R^2C}$$
$$Q=\sqrt{\frac{1}{R^2C}}$$
$$\boxed{Q=\frac{1}{R}\sqrt{\frac{L}{C}}}$$

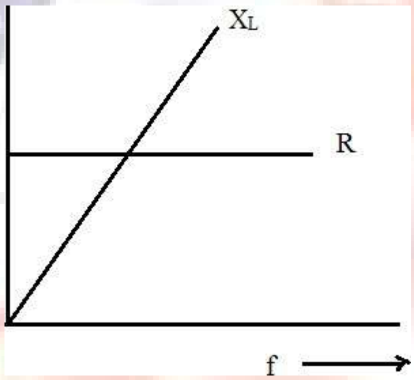
GraphicalMethod:→

(1) Resistanceisindependentoffrequency Itrepresentsastraightline.

(2) InductiveReactance $X_L = 2\pi fL$

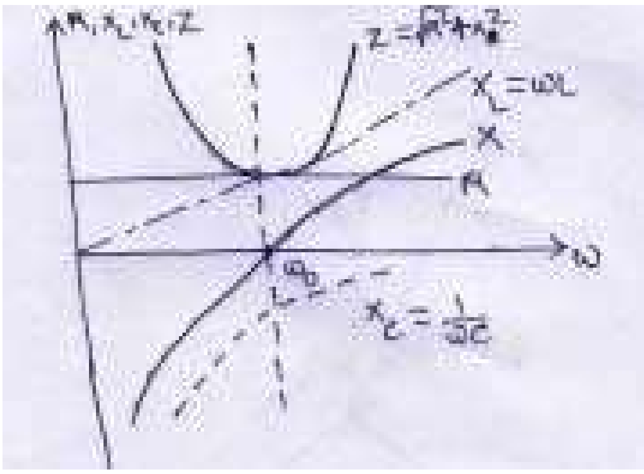
It is directly proportional to frequency. As the frequency increases , $X_L$increases

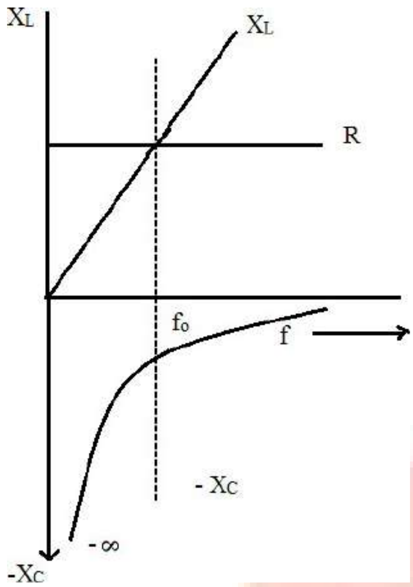
(3) CapacitiveReactance$X_C = \frac{1}{2\pi fC}$



It is inversely proportional to frequency. As the frequency increases, $X_C$decreases.

When frequency increases, $X_L$increases and $X_C$decreasesfromthe highervalue.



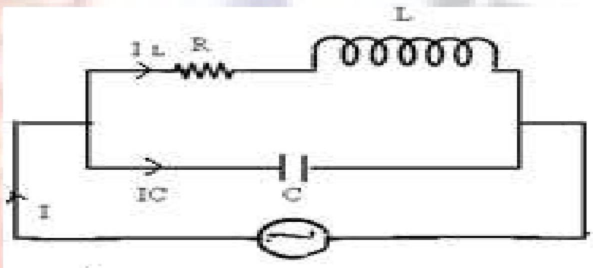


Atacertainfrequency. $X_L=X_C$

ThatparticularfrequencyisknownasResonantfrequency.

Variationofcircuitparameterinseriesresonance:

(2) Parallel Resonance :- Resonance willoccur when the reactive part of the line current is zero.



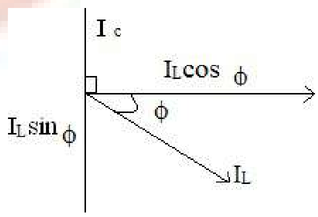
At resonance,

$I_C - I_L\sin\phi = 0$

$I_C = I_L\sin\phi$

$$\Rightarrow \frac{V}{X_C} = \frac{V}{\sqrt{R^2+X_L^2}}\sin\phi$$

$$\Rightarrow \frac{V}{X_C} = \frac{V}{\sqrt{R^2+X_L^2}} \times \frac{X_L}{\sqrt{R^2+X_L^2}}$$



$$\Rightarrow \frac{1}{X_C} = \frac{X_L}{R^2+X_L^2}$$

$$\Rightarrow R^2+X_L^2 = X_L.X_C$$

$$\Rightarrow Z^2 == X_L.X_C = W_0L \times \frac{1}{W_0C}$$

$$Z^2 = \frac{L}{C}$$

$$\Rightarrow R^2 + X_L^2 = \frac{L}{C}$$

$$\Rightarrow R^2 + (2\pi f_0 L)^2 = \frac{L}{C}$$

$$\Rightarrow R^2 + 4\pi^2 f_0^2 L^2 = \frac{L}{C}$$

$$\Rightarrow 4\pi^2 f_0^2 L^2 = \frac{L}{C} - R^2$$

$$\Rightarrow f_0^2 = \frac{1}{4\pi^2 f_0^2 L} = \left(\frac{L}{C} - R^2\right)$$

$$\Rightarrow f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

f_0=Resonantfrequencyinparallelcircuit.

Current at Resonance $= I_L \cos\phi$

$$= \frac{V}{\sqrt{R^2 + X_L^2}} \cdot \frac{R}{\sqrt{R^2 + X_L^2}}$$

$$= \frac{VR}{R^2 + X_L^2}$$

$$= \frac{VR}{Z^2}$$

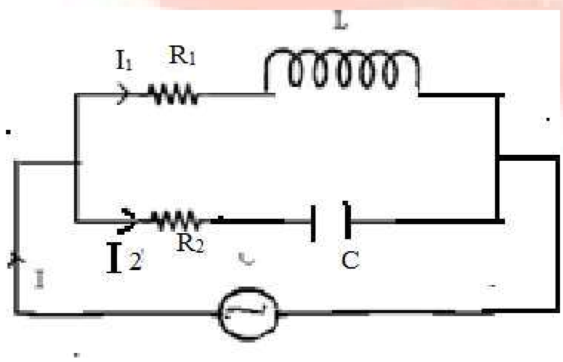
$$= \frac{VR}{L/C} = \frac{V}{L/RC}$$

$$= \frac{V}{\text{Dynamic Impedence}}$$

$L/RC \rightarrow$DynamicImpedanceofthe circuit.

or, dynamic impedances is defined as the impedance at resonance frequency in parallel circuit.

ParallelCircuit:→



Theparallelresonancecondition:

Whenthereactivepartofthelinecurrentiszero. The net reactance is zero.
Thelinecurrentwillbeminimum.
The power factor will be unity

Impedance $Z_1 = R_1 + jX_L$

$Z_2 = R_2 - jX_C$

Admittance $Y_1 = \frac{1}{Z_1} = \frac{1}{R_1 + jX_L}$

$= \frac{(R_1 + jX_L)}{(R_1 + jX_L)(R_1 - jX_L)}$

$= \frac{R_1 + jX_L}{R_1^2 + X_L^2}$

$Y_1 = \frac{R_1}{R_1^2 + X_L^2} - j\frac{X_L}{R_1^2 + X_L^2}$

Admittance $Y_2 = \frac{1}{Z_2} = \frac{1}{R_1 + jX_C}$

$= \frac{(R_2 + jX_C)}{(R2_1 - jX_C)(R_2 + jX_C)}$

$= \frac{R_2 + jX_L}{R_2^2 + X_C^2}$

$Y_2 = \frac{R_2}{R_2^2 + X_C^2} + j\frac{X_C}{R_2^2 + X_C^2}$

TotalAdmittanceAdmittance $\left(\frac{1}{Z}\right) = \frac{1}{Z_1} + \frac{1}{Z_2}$

$\Rightarrow Y = Y_1 + Y_2$

$\Rightarrow Y = \frac{R_1}{R_1^2 + X_L^2} - j\frac{X_L}{R_1^2 + X_L^2} + \frac{R2}{R_2^2 + X_C^2} + j\frac{X_C}{R_2^2 + X_C^2}$

$\Rightarrow Y = \frac{R_1}{R_1^2 + X_L^2} + \frac{R2}{R_2^2 + X_C^2} - j\left(\frac{X_L}{R_1^2 + X_L^2} - \frac{X_C}{R_2^2 + X_C^2}\right)$

At Resonance,

$\frac{X_L}{R_1^2 + X_L^2} - \frac{X_C}{R_2^2 + X_C^2} = 0$

$\rightarrow \frac{X_L}{R_1^2 + X_L^2} = \frac{X_C}{R_2^2 + X_C^2}$

$\Rightarrow X_L\left(R_2^2 + X_C^2\right) = X_C\left(R_1^2 + X_L^2\right)$

$\Rightarrow 2\pi fL\left(R_2^2 + \frac{1}{4\pi^2 f^2 C^2}\right) = \frac{1}{2\pi fC}\left(R_1^2 + 4\pi^2 f^2 L^2\right)$

$\Rightarrow 2\pi fLR_2^2 + \frac{L}{2\pi fC^2} = \frac{R_1^2}{2\pi fC} + \frac{2\pi fL^2}{C}$

$$\Rightarrow \frac{L}{2\pi f C^2} - \frac{R_2^2}{2\pi f C} = \frac{2\pi f L^2}{C} - 2\pi f L R_2^2$$

$$\Rightarrow \frac{1}{2\pi f C}\left(\frac{L}{C} - R_1^2\right) = 2\pi f L\left(\frac{L}{C} - R_2^2\right)$$

$$\Rightarrow 4\pi^2 f^2 LC = \frac{\frac{L}{C} - R_1^2}{\frac{L}{C} - R_2^2} = \frac{L - CR_1^2}{L - CR_2^2}$$

$$\Rightarrow 4\pi^2 f^2 = \frac{1}{LC}\left(\frac{L - CR_1^2}{L - CR_2^2}\right)$$

$$\Rightarrow f^2 = \frac{1}{4\pi^2 LC}\left(\frac{L - CR_1^2}{L - CR_2^2}\right)$$

$$\Rightarrow f = \frac{1}{2\pi\sqrt{LC}}\sqrt{\left(\frac{L - CR_1^2}{L - CR_2^2}\right)}$$

$$\Rightarrow f = \frac{1}{2\pi}\sqrt{\left(\frac{L - CR_1^2}{L^2C - LC^2R_2^2}\right)}$$

f is called Resonant frequency.

If $R_2 = 0$

Then $f = \frac{1}{2\pi}\sqrt{\frac{L - CR_1^2}{L^2C}}$

$$= \frac{1}{2\pi L}\sqrt{\frac{L - CR_1^2}{C}}$$

$$= \frac{1}{2\pi L}\sqrt{\frac{L}{C} - R_1^2}$$

$$= \frac{1}{2\pi}\sqrt{\frac{L}{L^2C} - \frac{R_1^2}{L^2}}$$

$$\boxed{f = \frac{1}{2\pi}\sqrt{\frac{L}{LC} - \frac{R_1^2}{L^2}}}$$

If R_1 and $R_2 = 0$, then

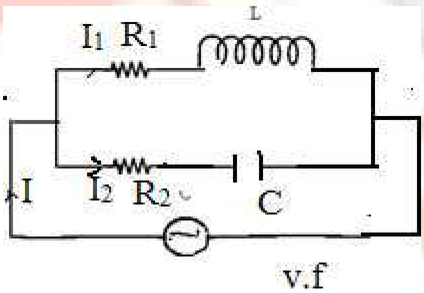
$$f = \frac{1}{2\pi}\sqrt{\frac{L}{L^2C}}$$

$$\boxed{f = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} = \frac{1}{2\pi\sqrt{LC}}}$$

Comparison of Series and Parallel Resonant Circuit:→

Item	Seriesckt(R-L-C)	Parallelckt(R–Land C)
❖ ImpedanceatResonance	Minimum	Maximum
❖ CurrentatResonance	Maximum=$\frac{V}{R}$	Minimum=$\frac{V}{(L/CR)}$
❖ EffectiveImpedance	R	$\frac{L}{CR}$
❖ P.f.at Resonance	Unity	Unity
❖ ResonantFrequency	$\frac{1}{2\pi\sqrt{LC}}$	$\frac{1}{2\pi}\sqrt{\frac{1}{LC}-\frac{R^2}{L^2}}$
❖ ItMagnifies	Voltage	Current
❖ Magnificationfactor	$\frac{WL}{R}$	$\frac{WL}{R}$

Parallelcircuit:→



$Z_1=R_1+jX_L=\sqrt{R_1^2+X_L^2}\angle\phi_1$

$Z_2=R_1-jX_C=\sqrt{R2_1^2+X_C^2}\angle-\phi_2$

$I_1=\frac{V}{Z_1\angle\phi_1}=\frac{V}{Z_1}\angle-\phi_1=I_1\angle-\phi_1$

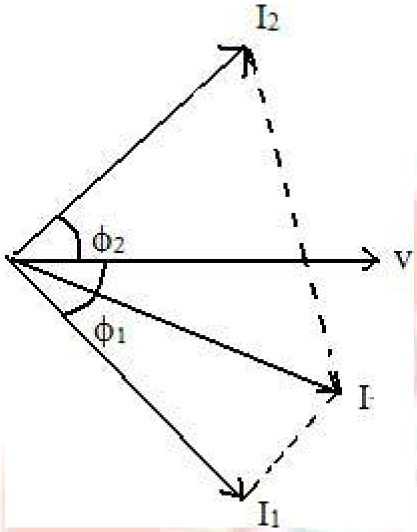
Where $\frac{V}{Z_1}=VY_1$

HereY$_1$→Admittanceofthecircuit

Admittanceisdefinedasthereciprocalofimpedence.

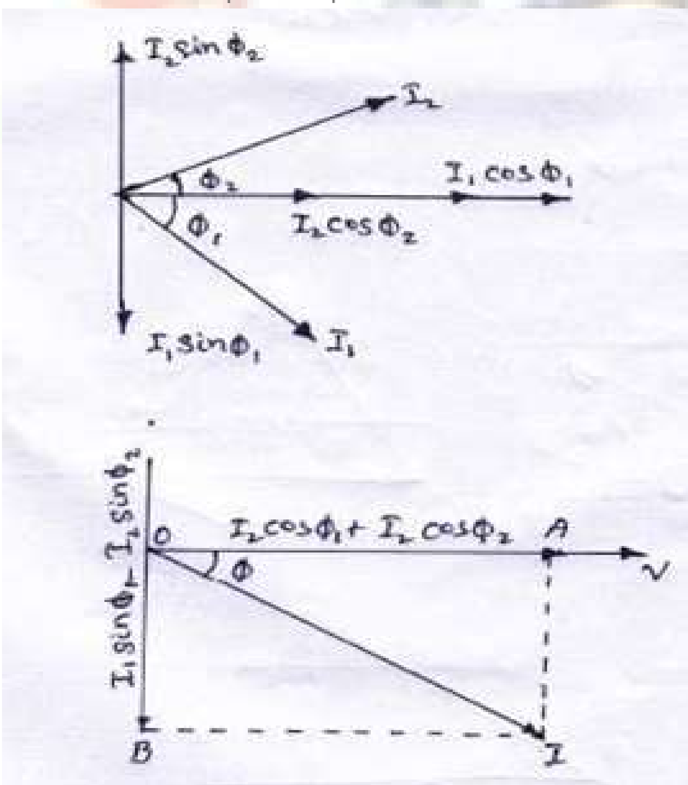
$$I_1 = VY_1 = \frac{V}{R_1 + jX_L}$$

$$I_2 = \frac{V}{Z_2\angle -\phi_2} = \frac{V}{Z_2}\angle\phi_2 = VY_2\angle\phi = I_2\angle\phi_2$$



$$I = \sqrt{I_1^2 + I_2^2 + 2I_1I_2\cos(\phi_1 + \phi_2)}$$

$$I = I_1\angle -\phi_1 + I_2\angle\phi_2$$



The resultant current "I" is the vector sum of the branch currents I_1 & I_2 can be found by using parallelogram low of vectors or resolving I_2 into their X

–andY-components(oractiveandreactivecomponentsrespectively)andthen by combining these components.

SumofactivecomponentsofI_1andI_2=$I_1\cos\phi_1+I_2\cos\phi_2$

SumofthereactivecomponentsofI_1andI_2=$I_2\sin\phi_2-I_1\sin\phi_1$

EXP–01:

A60Hzvoltageof230Veffectivevalueisimpressedonaninductanceof 0.265H

(i) Writethetimeequationforthevoltageandtheresultingcurrent.Letthe zero axis of the voltage wave be att= 0.

(ii) Showthevoltageandcurrentonaphasordiagram.

(iii) Findthemaximumenergystoredintheinductance.

Solution:-

$V_{max}=\sqrt{2}V=\sqrt{2}\times 230V$

f=60Hz, $W=2\pi f=2\pi\times 60=377 rad/s.$

$x_l=wl=377\times 0.265=100\Omega$

(i) Thetimeequationforvoltageis$V(t)=230\sqrt{2}\sin 377t.$

$I_{max}=V_{max}/x_l=230\sqrt{2}/100.=2.3\sqrt{3}$

$\phi=90^o(lag).$

∴Currentequationis.

$i(t)=2.3\sqrt{2}\sin(377t-\pi/2)$

or $=2.3\sqrt{2}\cos 377t$

(ii) Iti

(iii) or$E_{max}=\frac{1}{2}LI^2_{max}=\frac{1}{2}\times 0.265\times(2.3\sqrt{2})^2=1.4J$

Example-02:

The potential difference measured across a coil is 4.5 v, when it carries a direct current of 9 A. The same coil when carries an alternating current of 9A at 25 Hz, the potential difference is 24 v. Find the power and the power factor when it is supplied by 50 v, 50 Hz supply.

Solution:

LetRbethed.c.resistanceandLbeinductanceofthecoil.

$R=V/I=4.5/9=0.5\Omega$

Witha.c.currentof25Hz,z=V/1.

$\frac{24}{9}=2.66\Omega$

$x_l=\sqrt{Z^2-R^2}=\sqrt{2.66^2-0.5^2}$

$=2.62\Omega$

$x_l=2\pi\times25\times L$

$x_l=0.0167\Omega$

At50Hz

$x_l=2.62\times2=5.24\Omega$

$Z=\sqrt{0.5^2+5.24^2}$

$=5.06\ \Omega$

I=50/5.26 =9.5A

$P=I^2/R=9.5^2\times0.5=45$watt.

Example–03:

A50-µfcapacitorisconnectedacrossa230-v,50–Hzsupply. Calculate

(a) Thereactanceofferedbythecapacitor.

(b) Themaximumcurrentand

(c) Ther.m.svalueofthecurrentdrawnbythecapacitor.

Solution:

(a) $x_l=\frac{1}{wc}=\frac{1}{2\pi fe}=\frac{1}{2\pi\times50\times50\times10^{-6}}=63.6\Omega$

(c) Since230vrepresentsther.m.svalue

$\therefore I_{rms}=230/x_l=230/63.6=3.62A$

(b) $I_m=I_{r.m.s}\times\sqrt{2}=3.62\times\sqrt{2}=5.11A$

Example–04:

In a particularR –L series circuita voltageof 10v at50 Hzproduces a current of 700 mA. What are the values of R and L in the circuit ?

Solution:

(i) $Z=\sqrt{R^2+(2\pi\times50L)^2}$

$=\sqrt{R^2+98696L^2}$

$V=1z$

$10=700\times10^{-3}\sqrt{(R^2+98696L^2)}$

$\sqrt{(R^2+98696L^2)}=10/700\times10^{-3}=100/7$

$R^2+98696L^2=10000/49$-------------------------(I)

(ii) Inthesecondcase$Z=\sqrt{R^2+(2\pi\times75L)^2}$

$\therefore 10=500\times10^{-3}\sqrt{R^2+222066L^2)}=20$

$\sqrt{R^2+222066L^2)}=20$

$R^2+222066L^2=400$ ------------------------- (II)

SubtractingEa.(I)from(ii),weget,

$222066L^2-98696L^2=400-(10000/49)$

$\Rightarrow 123370L^2=196$

$\Rightarrow L^2=\dfrac{196}{123370}$

$\Rightarrow L=\sqrt{\dfrac{196}{123370}}=0.0398H$ = 40mH.

SubstitutingthisvalueofLinequation(ii)weget $R^2+222066L^2(0.398)^2=400$

$\Rightarrow R=6.9\Omega$.

Example–04:

A 20Ωresistor is connected in series with an inductor, a capacitor and an ammeter across a 25 –v, variable frequency supply. When the frequency is 400Hz, the current is at its Max^m value of 0.5 A and the potential difference across the capacitor is 150v. Calculate

(a) Thecapacitanceofthecapacitor.

(b) Theresistanceandinductanceoftheinductor.

Solution:

Sincecurrentismaximum,thecircuitisinresonance.

$x_l=V_C/1=150/0.5=300\Omega$

(a) $x_l=1/2\pi fe \Rightarrow 300=1/2\pi\times 400\times c$

$\Rightarrow c=1.325\times 10^{-6}f=1.325\mu f.$

(b) $x_l=x_l=150/0.5=300\Omega$

$2\pi\times 400\times L=300\Rightarrow$

L = 0.49H

(c) At resonance,

Circuitresistance=20+R

$\Rightarrow$ V/Z = 2510.5

$\Rightarrow R=30\Omega$

Exp.-05

An R-L-C series circuits consists of a resistance of 1000Ω, an inductance of 100MH an a capacitance of wμμf or 10PK

(ii) Thehalfpowerpoints.

Solution:

i) $fo=\dfrac{1}{2\pi\sqrt{10^{-1}\times 10^{-4}}}=\dfrac{10^6}{2\pi}=159KHz$

ii) $\phi = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{1000}\times\sqrt{\frac{10^{-1}}{10^{-11}}} = 100$

iii) $f_1 = fo - \frac{R}{4\pi l} = 159\times10^{3} - \frac{1000}{4\pi\times10^{-1}} = 158.2KHz$

$f_2 = fo - \frac{R}{4\pi l} = 159\times10^{-3} + \frac{1000}{4\pi\times10^{-1}} = 159.8KHz.$

Exp.-06

Calculatetheimpedanceoftheparallel–turnedcircuitasshowninfig. 14.52 at a frequency of 500 KHz and for band width of operation equal to 20 KHz. The resistance of the coil is 5Ω.

Solution:

At resonance, circuit impedance is L/CR. We have been given the valueof R but that of L and C has to be found from the given the value of R but thatof L and C has to be found from the given data.

$BW = \frac{R}{2\pi l}, 20\times10^{3} = \frac{5}{2\pi\times l} or l = 39 H_{\mu}$

$fo - \frac{1}{2\pi} = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} = \frac{1}{2\pi}\sqrt{\frac{1}{39\times10^{-6}C} - \frac{5^2}{(39\times10^{-6})^2}}$

$C = 2.6\times10^{-9}$

$Z = L/CR = 39\times10^{-6}/2.6\times10^{-9}\times5$

$= 3\times10^{3}\Omega$

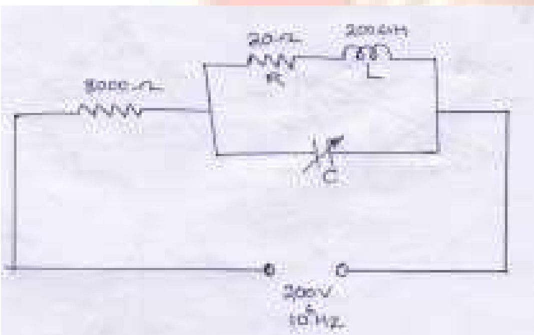
Example: A coil of resistance 20Ω and inductance of 200μH is in parallel with a variable capacitor. This combination is series with a resistor of 8000Ω.The voltage of the supply is 200V at a frequency of$10^6 H_Z$.Calculate

i) thevalueofCtogive resonance

ii) the Qofthe coil

iii) thecurrentineachbranchofthecircuitatresonance

Solution:



$X_L = 2\pi fL = 2\pi * 10^6 * 200 * 10^{-6} = 1256\Omega$

Thecoilisnegligibleresistanceincomparisontoreactance.

$$\boxed{f = \frac{1}{2\pi\sqrt{LC}}}$$

$$\boxed{10^6 = \frac{1}{2\pi\sqrt{200 * C * 10^{-6}}}}$$

ii) Q=$\frac{2\pi f L}{R} = 2\pi * 10^6 * 200 * \frac{10^{-4}}{20}$=62.8

iii) dynamicimpedanceofthecircuitZ=L/CR=200*10^{-6}/(125*10^{-12}*20)=80000Ω

totalZ=80000+8000=88000Ω

I=200/88000=2.27mA

p.d across tuned circuit=2.27*10^{-3}*80000=181.6Vcurrentthroughinductivebranch=

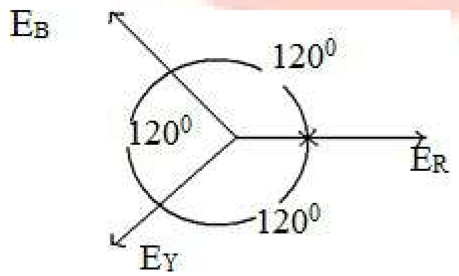
$\frac{181.6}{\sqrt{10^2+1256^2}} = 144.5mA$current through capacitor branch=

ωVC

=181.6*2π*10^6*125*10^{-12}=142.7mA

POLY-PHASECIRCUIT

Three-phasecircuitsconsistsofthreewindingsi.e.R.Y.B



$E_R=E_m\sin wt=E_m\angle 0$

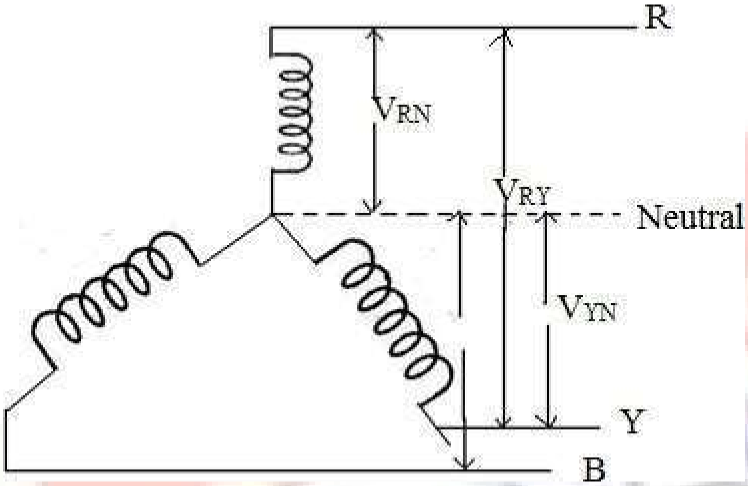
$E_Y=E_m\sin(wt-120)=E_m\angle -120$

$E_B=E_m\sin(wt-240)=E_m\angle -240=E_m\angle 120$

3-ϕCircuitaredividedintotwotypes

- StarConnection
- DeltaConnection

StarConnection:→



If three similar ends connected at one point, then it is known as star connected system.

The common point is known as neutral point and the wire taken from the neutral point is known as Neutral wire.

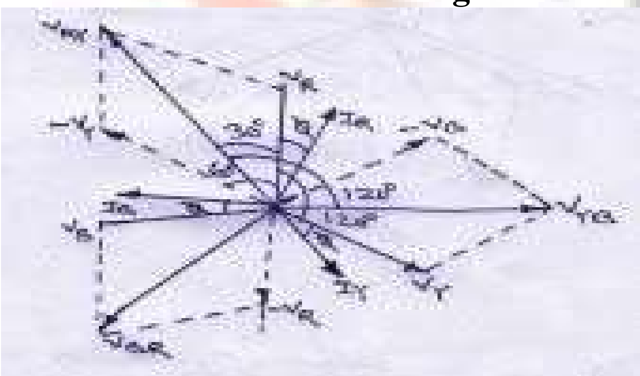
PhaseVoltage:→

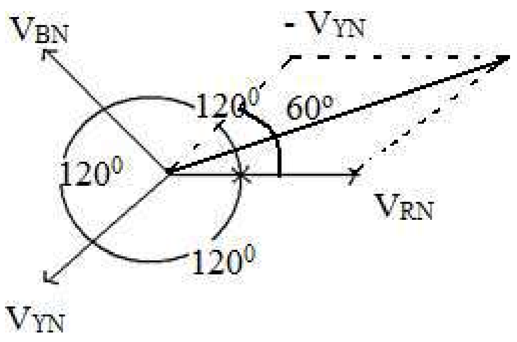
Itisthepotentialdifferencebetweenphaseand Neutral.

LineVoltage:→

ItisItisthepotentialdifferencebetweentwophases.

RelationBetweenPhaseVoltageandLineVoltage:→





Line Volatage $\vec{V}_{RY} = \vec{V}_{RN} - \vec{V}_{YN}$ $V_L = \sqrt{V_{RN}^2 + V_{YN}^2 - 2V_{RN}V_{YN}\cos 60^o}$

$$= \sqrt{V_{Ph}^2 + V_{ph}^2 - 2V_{ph}V_{ph} \times \frac{1}{2}}$$

$$= \sqrt{3V_{Ph}^2} = \sqrt{3}V_{Ph}$$

$$V_L = \sqrt{3}V_{Ph}$$

Since in a balanced B–phase circuit $V_{RN} = V_{YN} = V_{BN} = V_{ph}$

Relation Between Line current and Phase Current:-

In case of star connection system the leads are connected in series with each phase

Hence the line current is equal to phase current $I_L = I_{ph}$

Power in 3-Phase circuit:-

$P = V_{ph} I_{ph} \cos\phi$ per phase

$= 3V_{ph} I_{ph} \cos\phi$ for 3 phase

$= 3 \frac{V_L}{\sqrt{3}} I_L \cos\phi \; (\because V_L = \sqrt{3}V_{ph})$

$P = \sqrt{3} V_L I_L \cos\phi$

Summaries in star connection:

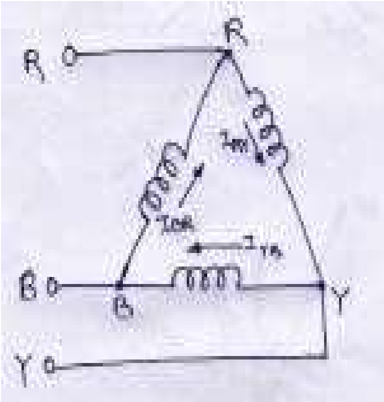
i) The line voltages are 120^0 apart from each other.

ii) Line voltages are 30^0 ahead of their respective phase voltage.

iii) The angle between line currents and the corresponding line voltage is $30 + \varphi$

iv) The current in line and phase are same.

Delta Connection:-



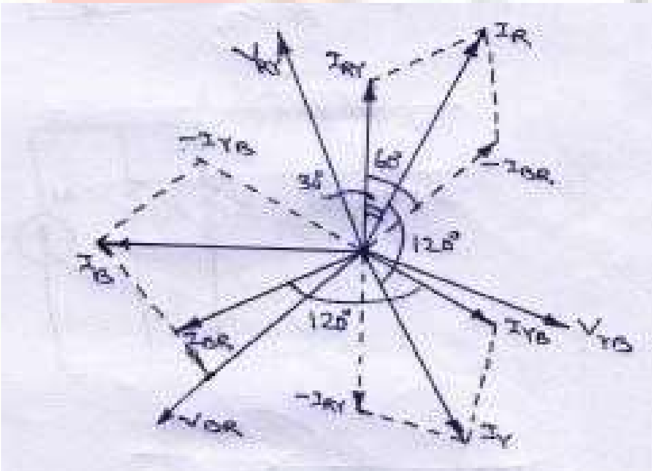
If the dissimilar ends of the closed mesh then it is called a Delta Connected system

RelationBetweenLineCurrentandPhaseCurrent:-

Line Currentinwire – 1= $\vec{i_R} - \vec{i_Y}$

LineCurrentin wire -2= $\vec{i_Y} - \vec{i_B}$

Line Currentinwire–3= $\vec{i_B} - \vec{i_R}$



$$\vec{I_L} = \vec{I_R} - \vec{I_Y}$$

$$= \sqrt{I_R^2 + I_Y^2 - 2I_R I_Y \cos 60^0}$$

$$= \sqrt{I_{ph}^2 + I_{ph}^2 - 2I_{ph} I_{ph} \times \frac{1}{2}}$$

$$= \sqrt{3I_{ph}^2}, I_L = \sqrt{3I_{ph}^2}$$

$$I_L = \sqrt{3} I_{ph}$$

RelationBetweenLineVoltage&PhaseVoltage:→

$V_L = V_{ph}$

Power== $\sqrt{3} V_L I_L \cos\phi$

Summariesindelta:

i) Linecurrentsare 120^o apartfromeachother.

ii) Linecurrentsare 30^o behindtherespectivephasecurrent.

iii) Theanglebetweenthelinecurrentsandcorrespondinglinevoltagesis30+φ

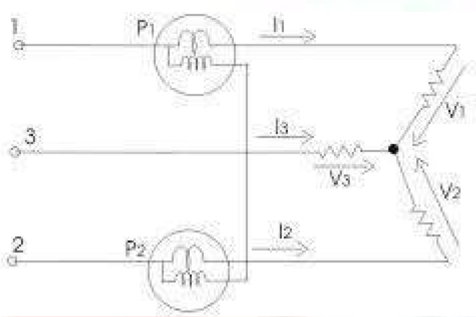
MeasurementofPower:→

(1) Bysinglewatt-metermethod

(2) ByTwo-wattmeterMethod

(3) ByThree-wattmeterMethod

MeasurementofpowerByTwoWattMeterMethod :-



PhasorDiagram:-

LetV$_R$,V$_Y$,V$_B$arether.m.svalueof3-ϕvoltagesandI$_R$,I$_Y$,I$_B$arether.m.s. values of the currents respectively.

CurrentinR-phasewhichflowsthroughthecurrentcoilofwatt-meter $W_1 = I_R$

And $W_2=I_Y$

Potentialdifference acrossthevoltagecoilof$W_1=\vec{V}_{RB}=\vec{V}_R-\vec{V}_B$

And$W_2=\vec{V}_{YB}=\vec{V}_Y-\vec{V}_B$

Assumingtheloadisinductivetypewatt-meterW_1reads.

$$W_1=V_{RB}I_R\cos(30-\phi)$$

$$W_1=V_LI_L\cos(30-\phi) \text{ -------------------------------- } (1)$$

WattmeterW_2reads

$$W_2=V_{YB}I_Y\cos(30+\phi)$$

$$W_2=V_LI_L\cos(30+\phi) \text{ ---------------------- } (2)$$

$$W_1+W_2=V_LI_L\cos(30-\phi)+V_LI_L\cos(30+\phi)$$

$$=V_LI_L[\cos(30-\phi)+V_LI_L\cos(30+\phi)]$$

$$=V_LI_L(2\cos30^o\cos\phi)$$

$$=V_LI_L(2\times\frac{\sqrt{3}}{2}\cos\phi)$$

$$W_1+W_2=\sqrt{3}V_LI_L\cos\phi \text{ ---------------------- } (3)$$

$$W_1-W_2=V_LI_L[\cos(30-\phi)-\cos(30+\phi)$$

$$=V_L I_L(2\sin 30^o \sin\phi)$$
$$=V_L I_L\left(2\times\frac{1}{2}\times\sin\phi\right)$$
$$W_1-W_2=V_L I_L\sin\phi$$
$$\frac{W_1-W_2}{W_1+W_2}=\frac{V_L I_L\sin\phi}{\sqrt{3}V_L I_L\cos\phi}$$
$$\frac{1}{\sqrt{3}}=\tan\phi$$
$$\Rightarrow\tan\phi=\sqrt{3}\left(\frac{W_1-W_2}{W_1+W_2}\right)$$
$$\Rightarrow\phi=\tan^{-1}\sqrt{3}\left(\frac{W_1-W_2}{W_1+W_2}\right)$$

Variationinwattmeterreadingwithrespecttop.f:

Pf	W_1reading	W_2reading
φ=0,cosφ=1	+veequal	+veequal
φ=60,cosφ=0.5	0	+ve
φ=90,cosφ=0	-ve,equal	+veequal

Exp. :01

A balanced star – connected load of (8+56). Per phase is connected to a balanced 3-phase 100-v supply. Find the cone current power factor, power and total volt-amperes.

Solution:

$Z_{ph}=\sqrt{8^2+6^2}=10\Omega$

$V_{ph}=400/\sqrt{3}=23/v$

$I_{ph}=V_{ph}/Z_{ph}=231/10=23.1A$

i) $I_L=Z_{ph}=23.1A$

ii) P.f.=$\cos\theta=R_{ph}/z_{ph}=8/10=0.8$(lag)

iii) $PowerP=\sqrt{3}V_L I_L\cos\theta$

$=\sqrt{3}\times 400\times 23.1\times 0.8$

=12,800watt.

iv) Totalvoltamperes=$\sqrt{3}V_L I_L$

$=\sqrt{3}\times 400\times 23.1$

=16,000VA.

Exp.-02

Phase voltage and current of a star-connected inductive load is 150V and 25A. Power factor of load as 0.707 (Lag). Assuming that the system is 3-wireand power is measured using two watt meters, find the readings of watt meters. **Solution :**

$V_{ph} = 150V$

$V_L = \sqrt{3} \times 150$

$I_{ph} = I_L = 25A$

Total power $= \sqrt{3}\, V_L I_L \cos\phi = \sqrt{3} \times 150 \times \sqrt{3} \times 25 \times 0.707 = 7954$watt.

$W_1 + W_2 = 7954.00$, $\cos\phi = 0.707$

$\phi = \cos^{-1}(0.707) = 45°$, $\tan 45° = 1$

Nowforalaggingpowerfactor,

$\tan\phi = \sqrt{3}(W_1 - W_2)/(W_1 + W_2)$

$\Rightarrow 1 = \sqrt{3}\left\{\frac{(W_1 - W_2)}{7954}\right\}$

$\therefore (W_1 - W_2) = 4592w$

From(i)and(ii)above,we get

$W_1 = 6273w$ $W_2 = 1681w$

TRANSIENTS

Whenever a network containing energy storage elements such as inductor or capacitor is switched from one condition to another,either by change in applied source or change in network elements,the response current and voltage change from one state to the other state.Thetimetakentochangefromaninitial steadystatetothefinalsteadystateisknown as the *transient period*.This response is known as *transient response* or *transients.*The response of the network after it attains a final steady value is independent of time and is calledthesteady-stateresponse.Thecompleteresponseofthenetworkisdeterminedwith the helpofa differential equation.

STEADYSTATEANDTRANSIENT RESPONSE

In a network containing energy storage elements, with change in excitation, the currents and voltages in the circuit change from one state to other state. The behaviour of the voltage orcurrent when it is changedfrom one state toanotheris calledthe transient state. The time taken for the circuit to change from one steady state to another steady state is called the transient time. The application of KVL and KCL to circuits containing energy storageelementsresultsindifferential,ratherthanalgebraicequations.whenweconsidera circuit containing storage elements which are independent of the sources, the response depends upon the nature of the circuit and is called natural response. Storage elements deliver their energy to the resistances. Hence, the response changes, gets saturated after some time,and is referred to asthe transient response. When we consider a source acting on a circuit, the response depends on the nature of the source or sources.This response is called forced response. In other words,the complete response of a circuit consists of two parts; the forced response and the transient response. When we consider a differential equation, the complete solution consists of two parts: the complementary function and the particularsolution.Thecomplementaryfunctiondiesoutaftershortinterval,andisreferred to as the transient response or source free response. The particular solution is the steady state response, or the forced response. The first step in finding the complete solution of a circuit is to form a differential equation for the circuit. By obtaining the differentialequation, several methods can be used to find out the complete solution.

DCRESPONSEOFANR-LCIRCUIT

Consideracircuitconsistingofaresistanceandinductanceasshowninfigure.Theinductor in the circuit is initially uncharged and is in series with the resistor.When the switch S is closed ,we can find the complete solution for the current.Application of kirchoff’s voltage law to the circuit results in the following differential equation.

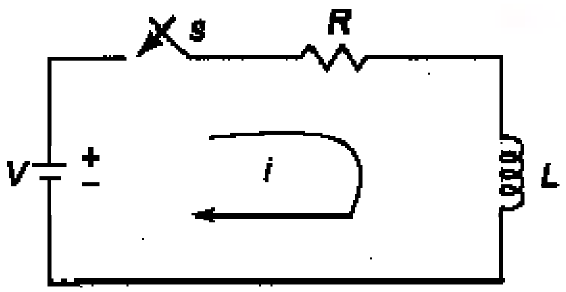


Figure 1.1

V = Ri + L$\frac{di}{dt}$ 1.1

Or$\frac{di}{dt}$+$\frac{R}{L}$i=$\frac{V}{L}$.. 1.2

Intheabove equation , the currentI is the solution to be found and V is the applied constant voltage.ThevoltageVisappliedtothecircuitonlywhentheswitchS isclosed.Theaboveequation is a linear differential equation of first order.comparing it with a non-homogenious differential equation

$\frac{dx}{dt}$+ P x = K 1.3

whosesolutionis

X=$e^{-pt}\int Ke^{+pt}$dt+c.....e^{-Pt}...1.4

Wherecisanarbitraryconstant.Inasimilarway,wecanwritethecurrentequationas

$$i=ce^{-\left(\frac{R}{L}\right)t}+e^{-\left(\frac{R}{L}\right)t}\int\frac{V}{L}e^{\left(\frac{R}{L}\right)t}dt$$

Hence,$i=ce^{-\left(\frac{R}{L}\right)t}+\frac{V}{R}$.. 1.5

To determine the value of c in equation c , we use the initial conditions .In the circuit shown in Fig.1.1,theswitchsis closed at t=0.att=0-,i.e.just beforeclosing theswitchs,thecurrent in the inductoriszero.Sincetheinductordoesnotallowsuddenchangesincurrents,att=o+ just after the switch is closed,the current remains zero.

Thus att=0,i=0

Substitutingtheaboveconditioninequationc,wehave 0 =

c +$\frac{V}{R}$

Substitutingthevalueofcinequationc,weget

$$i=\frac{V}{R}-\frac{V}{R}e^{\frac{-Rt}{L}}$$

$$i=\frac{V}{R}\left(1-e^{\frac{-Rt}{L}}\right)$$

i =I_0(1-$e^{\frac{-Rt}{L}}$)(where$I_0=\frac{V}{R}$)

i=I_0(1-$e^{\frac{-t}{\tau}}$)(where $\tau = Time\,constant = \frac{L}{R}$)... 1.6

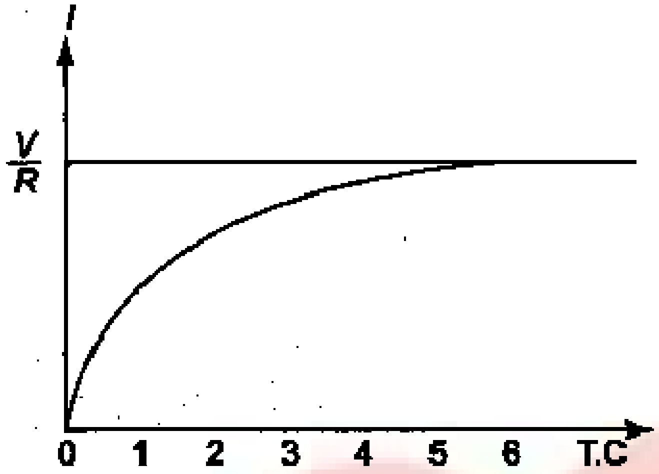


Figure1.2

Equationdconsistsoftwoparts,thesteadystatepart(I_o =V/R)and thetransientpart $I_o e^{\frac{-Rt}{L}}$.

WhenswitchSisclosed ,theresponsereachesasteadystate valueaftera timeintervalas shown in figure 1.2.

Here the transition period isdefined as the timetaken for the current toreach its final or stedy state value from its initial value.In the transient part of the solution, the quantityL/Ris importantindescribingthecurvesinceL/Risthetimeperiodrequired for the current to reach its initial value of zero to the final value I_o=V/R. The time constant of a function $I_o e^{\frac{-Rt}{L}}$ is thetimeatwhich theexponentofeisunity,wheree is the base of the natural logarithms.The term L/R is called the time constant and is denoted by τ .

So,$\tau = \frac{L}{R}$ sec

Hence,thetransientpartofthesolutionis

$$i = -\frac{V}{R}e^{\frac{-Rt}{L}} \qquad = \frac{V}{R}e^{\frac{-t}{\tau}}$$

AtoneTimeconstant,thetransienttermreaches36.8percentofitsinitial value.

$$i(\tau) = -\frac{V}{R}e^{\frac{-\tau}{\tau}} \quad = -\frac{V}{R}e^{-1} \quad = -0.368\,\frac{V}{R}$$

Similarly,

$$i(2\tau) = -\frac{V}{R}e^{-2} \quad = -0.135\,\frac{V}{R}$$

$$i(3\tau) = -\frac{V}{R}e^{-3} \quad = -0.0498\,\frac{V}{R}$$

$$i(5\tau) = -\frac{V}{R}e^{-5} \quad = -0.0067\,\frac{V}{R}$$

After5TCthetransientpartreachesmorethan99percentofitsfinal value.

InfigureAwecan findoutthevoltagesandpowersacrosseachelementbyusingthecurrent.

Voltageacrosstheresistoris

$$v_R = Ri = R \times \frac{V}{R}(1-e^{\frac{-Rt}{L}})$$

Hence, $v_R = V\,(1-e^{\frac{-Rt}{L}})$

Similarly,thevoltageacrosstheinductanceis $v_L =$

$$L\frac{di}{dt} = L\,\frac{V}{R}\times\frac{R}{L}e^{\frac{-Rt}{L}} = Ve^{\frac{-Rt}{L}}$$

TheresponsesareshowninFigure1.3.

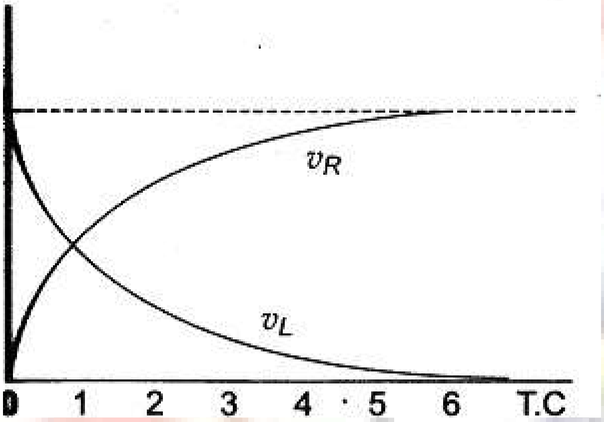


Figure1.3

Powerintheresistoris

$$P_R = v_R i = V(1-e^{\frac{-Rt}{L}})(1 - e^{\frac{-Rt}{L}}) \times \frac{V}{R} 2e^{\frac{-Rt}{L}}) +$$

$$= \frac{V^2}{R}\ (1- \ e^{\frac{-2Rt}{L}}$$

Powerintheinductoris

$$P_L = v_L i = Ve^{\frac{-Rt}{L}} \times \frac{V}{R}\ (1 - e^{\frac{-Rt}{L}})$$

$$= \frac{V^2}{R}(e^{\frac{-Rt}{L}} - e^{\frac{-2Rt}{L}})$$

Theresponsesareshowninfigure1.4.

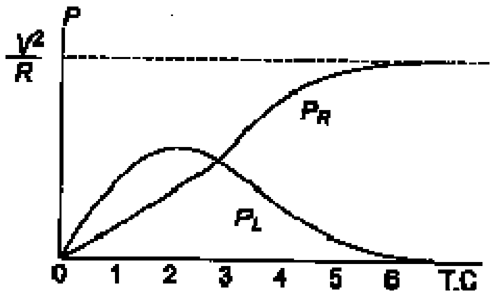


Figure1.4

Problem:1.1

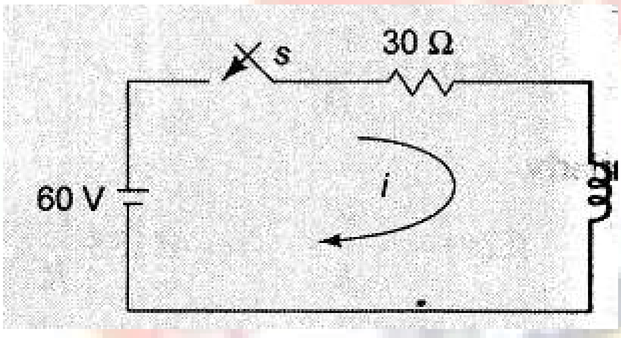


Figure1.5

AseriesR-LcircuitwithR=30Ωand L= 15 Hhasaconstant voltageV=50Vappliedatt=0as shown inFig.1.5 .determinethecurrent i,thevoltageacrossresistorandacrossinductor. Solution :

ByapplyingKirchoff'svoltageLaw,we get

$15\frac{di}{dt}+30i=60$

$=> \frac{di}{dt}+2i=4$

Thegeneralsolutionforalineardifferentialequationis i=c

$e^{-pt}+e^{-pt}\int Ke^{pt}dt$

where P=2,K=4

puttingthe valuesi=c

$e^{-2t}+e^{-2t}\int 4e^{2t}dt$

$=>i=ce^{-2t}+2$

Att=0,theswitchs isclosed.

Sincetheinductorneverallowssudden changein currents.At t=0^+thecurrent in thecircuit is zero. Therefore at t=0^+, i =0

=>0=c+ 2

=>c =- 2

Substitutingthevalueofcinthecurrentequation,wehave

i=2(1-e^{-2t})A

voltageacrossresistor(V_R)=iR=2(1-e^{-2t})x30=60(1-e^{-2t}) v

voltageacrossinductor(V_L)= L$\frac{di}{dt}$=15× $\frac{d}{dt}$2(1-e^{-2t})=30×2e^{-2t}v= $60e^{-2t}$

DCRESPONSEOFANR-CCIRCUIT

Consideracircuitconsistingofaresistanceand capacitanceasshowninfigure.The capacitorin the circuitisinitiallyunchargedandisinserieswiththeresistor.WhentheswitchSisclosed at t=0 , we can find the complete solution for the current.Application of kirchoff's voltage law to the circuit results in the following differential equation.

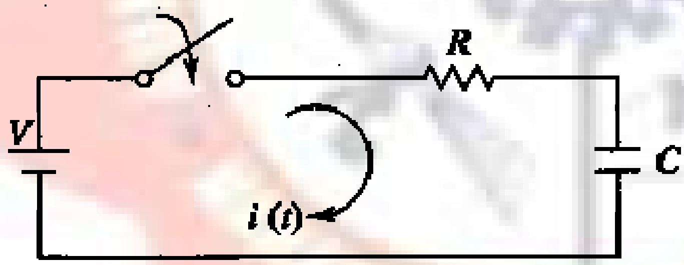


Figure1.6

V=Ri+$\frac{1}{C}\int i\,dt$..1.7

Bydifferentiatingtheaboveequation,weget

0=R $\frac{di}{dt}$ + $\frac{i}{C}$i 1.8

Or

$\frac{di}{dt}$ + $\frac{1}{RC}$ i=0 ....1.9

Equationcisalineardifferentialequationwithonlythecomplementaryfunction.Theparticular solution for the above equationis zero. The solution for this type of differential equationis

$i = ce^{-\left(\frac{t}{RC}\right)}$ 1.10

To determine the value of c in equation c , we use the initial conditions .In the circuit shown in Fig.theswitchsisclosed att=0.Sincethecapacitordoesnot allow suddenchangesinvoltage,it will act as a short circuitat t=o+ just after the switch is closed.

Sothecurrentinthecircuit att=0+is $\frac{V}{R}$ Thus at

t = 0,the current i = $\frac{V}{R}$

Substitutingtheaboveconditioninequationc,wehave $\frac{V}{R}$ = c

Substitutingthevalueofcinequationc,weget

$i = \frac{V}{R} e^{-\left(\frac{t}{RC}\right)}$ 1.11

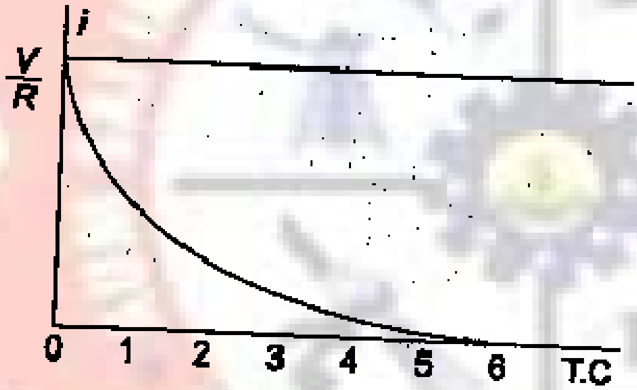


Figure1.7

WhenswitchSisclosed,theresponsedecaysasshowninfigurre. The

term RCis called the time constant and is denoted by τ .

So,τ=RCsec

After5 TCthecurvereaches99percentofitsfinalvalue.

InfigureAwecanfindoutthevoltageacrosseachelementbyusingthecurrentequation. Voltage

across the resistor is

$$v_R = Ri = R \times \frac{V}{R} e^{\frac{-t}{RC}}$$

Hence, $v_R = V e^{\frac{-t}{RC}}$

Similarly,voltageacrossthecapacitoris v_C

$= \frac{1}{C}\int i\, dt$

$= \frac{1}{C}\int \frac{V}{R} e^{\frac{-t}{RC}}\, dt$

$=-\left(\frac{V}{RC} \times RC\, e^{\frac{-t}{RC}}\right)+c$

$=-Ve^{\frac{-t}{RC}}+c$

Att=0,voltageacrosscapacitoriszero

So, c = V

And

$V_C = V\left(1 - e^{\frac{-t}{RC}}\right)$

Theresponsesareshownin Figure1.8.

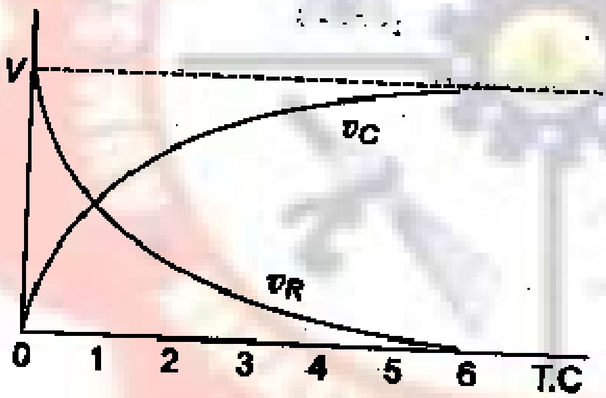


Figure1.8

Power in the resistor is

$P_R = v_R i = Ve^{\frac{-t}{RC}} \times \frac{V}{R} e^{\frac{-t}{RC}}$

$= \frac{V^2}{R} e^{\frac{-2t}{RC}}$

Powerinthecapacitoris P_C

$= v_C i = V\left(1- e^{\frac{-t}{RC}}\right) \frac{V}{R} e^{\frac{-t}{RC}}$

$=\frac{V^2}{R}(e^{\frac{-t}{RC}}-e^{\frac{-2t}{RC}})$

Theresponsesareshowninfigure1.9.

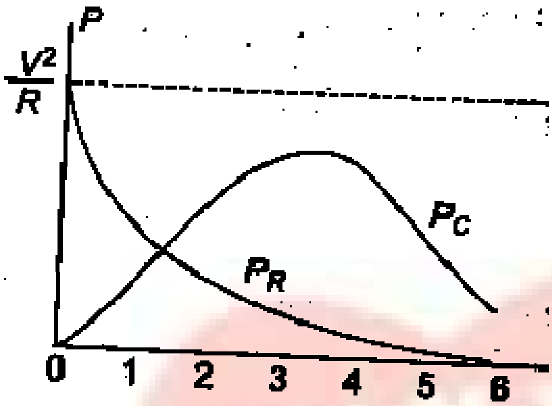


Figure1.9

Problem:1.2

A series R-C circuitwithR=10Ωand C=0.1 F has aconstant voltageV=20V appliedatt=0 as shown in Fig. determine the current i, the voltage across resistor and acrosscapacitor.

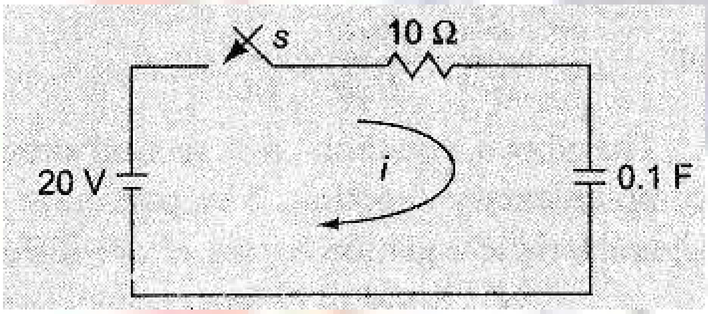


Figure1.10

Solution:

ByapplyingKirchoff'svoltageLaw,we get

$10i + \frac{1}{0.1}\int i\,dt = 20$

Differentiatingw.r.t.tweget

$10\frac{di}{dt}+\frac{i}{0.1}=0$

$=> \frac{di}{dt} + i = 0$

Thesolutionforaboveequation is

$i = ce^{-t}$

At t=0, the switch s is closed.

Since the capacitor never allows sudden change in voltages. At t=0^+ the current in the circuit is i = V/R=20/10 =2 A

.Therefore at t=0, i=2A

=>the current equation is $i=2e^{-t}$

voltage across resistor(V_R)=iR=$2e^{-t}$x10=$20e^{-t}$v

voltage across capacitor(V_C)= V$(1 - e^{\frac{-t}{RC}})$=20(1-e^{-t})V

DC RESPONSE OF AN R-L-C CIRCUIT

Consider a circuit consisting of a resistance, inductance and capacitance as shown in figure.The capacitor and inductor in the circuit is initially uncharged and are in series with the resistor.When the switch S is closed at t=0 , we can find the complete solution for the current.Application of kirchoff's voltage law to the circuit results in the following differential equation.

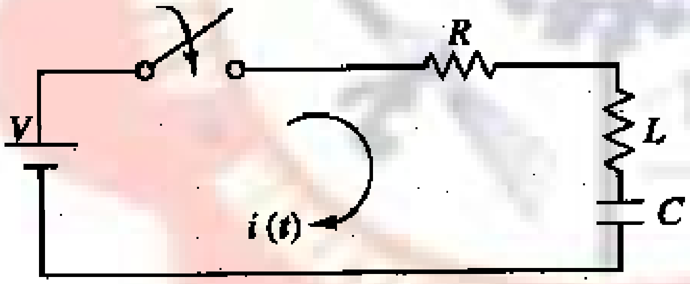


Figure1.11

$V = Ri + L\frac{di}{dt} + \frac{1}{C}\int i\,dt$ 1.12

By differentiating the above equation, we get

$0 = R\frac{di}{dt} + L\,d^2i/dt^2 + \frac{1}{C}i$ = ... 1.13

Or

$d^2i/dt^2 + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}\,i = 0$ 1.14

The above equation c is a second order linear differential equation with only the complementary function.Theparticularsolutionfortheaboveequationiszero.Thecharacteristicsequationforthis type of differential equationis

$D^2+\frac{R}{L}D+\frac{1}{LC}=0$ 1.15

Therootsofequation1.15are

$D_1, D_2 = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$

Byassuming$K_1=-\frac{R}{2L}$and$K_2=\sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$

$D_1 = K_1 + K_2$and$D_2 = K_1 - K_2$

HereK_2maybepositive,negativeorzero.

Case I :K_2 is Positive $\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$

Then,therootsareReal and Unequaland giveanoverdampedResponseasshowninfigure 1.12.

The solution for theaboveequationis:i $=C_1e^{(K_1+K_2)t}+C_2e^{(K_1-K_2)t}$

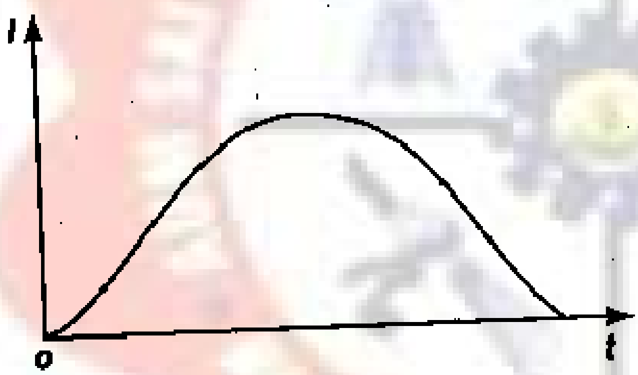


Figure1.12

Case II :K_2 is Negative $\left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$

Then,therootsareComplexConjugate,andgiveanunder-dampedResponseasshownin figure1.13.

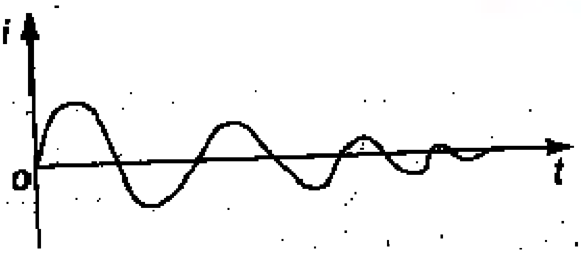


Figure1.13

Thesolutionfortheaboveequationis:i=$e^{K_1t}(C_1 \cos K_2t + C_2 \sin K_2t)$Case III :

Then,theK_2 is zero,$\left(\frac{R}{2L}\right)^2 = \frac{1}{LC}$ and give an Critically-damped Responseasshowninfigure1.14.

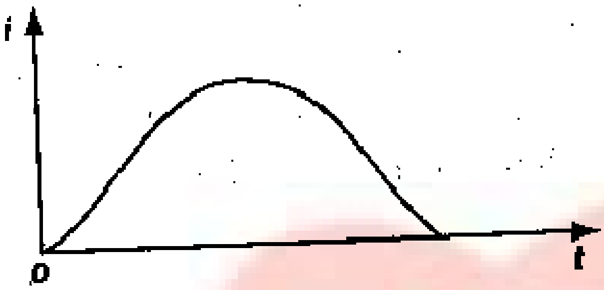


Figure1.14

Thesolutionfortheaboveequationis:i=$e^{K_1t}(C_1 + C_2t)$

Problem : 1.3

AseriesR-L-CcircuitwithR=20Ω,L=0.05HandC= 20 μFhasaconstantvoltageV=100 V appliedatt=0asshowninFig.determinethetransient currenti.

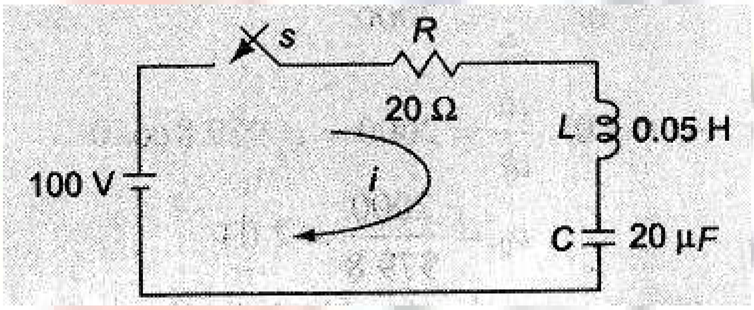


Figure1.15

Solution:

ByapplyingKirchoff'svoltageLaw,we get

$100 = 30i + 0.05\frac{di}{dt} + \frac{1}{20\times10^{-6}}\int i\,dt$

Differentiatingw.r.t.twe get

$0.05\,d^2i/dt^2 + 20\,\frac{di}{dt} + \frac{1}{20\times10^{-6}}\,i = 0$

$\Rightarrow d^2i/dt^2 + 400 \frac{di}{dt} + 10^6 i = 0$

$\Rightarrow (D^2 + 400D + 10^6)i = 0$

Therootsofequationare

$D_1, D_2 = -\frac{400}{2} \pm \sqrt{\left(\frac{400}{2}\right)^2 - 10^6}$

$= -200 \pm \sqrt{(200)^2 - 10^6}$

$D_1 = -200 + j979.8$

$D_2 = -200 - j979.8$

Thereforethecurrent

$i = e^{+K_1 t}[C_1 \cos K_2 t + C_2 \cos K_2 t]$

$i = e^{-200t}[C_1 \cos 979.8t + C_2 \sin 979.8t]$ A

Att=0,theswitchs isclosed.

Sincetheinductor neverallows sudden changein currents.Att=0^+thecurrent in thecircuit is zero. Therefore at t=0^+, i =0

$\Rightarrow i = 0 = (1)[C_1 \cos 0 + C_2 \sin 0]$

$\Rightarrow C_1 = 0$ andi= $e^{-200t}[C_2 \sin 979.8t]$A

Differentiatingw.r.t.twe get

$\frac{di}{dt} = C_2[e^{-200t} 979.8 \cos 979.8t + e^{-200t}(-200)\sin 979.8t]$

Att=0,thevoltageacrosstheinductoris100V $\Rightarrow L\frac{di}{dt}$

$= 100$ or $\frac{di}{dt} = 2000$

Att=0,$\frac{di}{dt} = 2000 = C_2 979.8 \cos 0$

$\Rightarrow C_2 = \frac{2000}{979.8} = 2.04$

Thecurrentequationis

i= $e^{-200t}(2.04\sin 979.8t)\,A$

ANALYSISOFCIRCUITSUSINGLAPLACETRANSFORMTE CHNIQUE

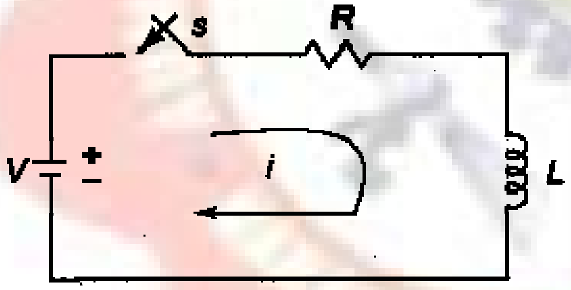
TheLaplace transform is a powerful Analytical Techniquethat is widely used to study the behaviorofLinear,Lumpedparametercircuits.LaplaceTransformconvertsatimedomain function f(t) to a frequency domain function F(s) and also Inverse Laplace transformation converts the frequency domain function F(s) back to a time domain function f(t).

L{f(t)}=F(s)= $\int_{-\infty}^{\infty} e^{-st}$f(t)dt...LT1

L^{-1}{ F(s)} = f(t) = $\frac{1}{2\pi j}\int_{-j}^{+j} F(s)\, e^{st}$ds..LT2

DCRESPONSEOFANR-LCIRCUIT(LT Method)

Let usdeterminethesolutionofthefirst order differential equationgivenbyequationAwhich is for the DC response of a R-L Circuit under the zero initial conditioni.e. current is zero, i=0 at t= 0^- and hence i=0at t= 0^+ in thecircuit in figure A bythe property of Inductance not allowing the current to change as switch is closed at t=0.



FigureLT1.1

V=Ri+L$\frac{di}{dt}$..LT1.1

TakingtheLaplaceTransformofbothesidesweget,

$\frac{V}{s}$=R I(s) + L [s I(s) −I(0)]..LT1.2

=> $\frac{V}{s}$=R I(s) + L [s I(s)] (I(0)=0:zeroinitialcurrent)

=> $\frac{V}{s}$= I(s)[R +L s]

=>I(s)= $\frac{V}{s[R+Ls]}$...LT1.3

TakingtheLaplaceInverseTransformofbothsidesweget,

$=> L^{-1}\{I(s)\}=i(t) = L^{-1}\{\frac{V}{s[R+Ls]}\}$

i(t)= $L^{-1}\{\frac{V/L}{s[R/L+s]}\}$(DividingthenumeratoranddenominatorbyL)

putting $\alpha = R/L$ weget

i(t)= $L^{-1}\{\frac{V/L}{s[s+\alpha]}\}=L^{-1}\{\frac{V}{L}(\frac{1}{s}-\frac{1}{(s+\alpha)})\frac{1}{\alpha}\}$

i(t)= $L^{-1}\{\frac{V}{L}(\frac{1}{s}-\frac{1}{(s+R/L)})\frac{L}{R}\}$(againputtingbackthevalueof α)

i(t)= $L^{-1}\{\frac{V}{R}(\frac{1}{s}-\frac{1}{(s+R/L)})\} = \frac{V}{R}(1-e^{\frac{-Rt}{L}})=I_o(1-e^{\frac{-Rt}{L}})$ (where $I_o = \frac{V}{R}$)

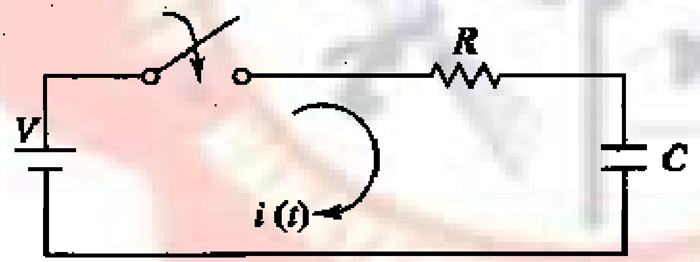
i(t)= $I_o(1-e^{\frac{-t}{\tau}})$ (where $\tau = Time\ constant = \frac{L}{R}$)..LT1.4

It canbeobserved that solutionfori(t)asobtainedbyLaplaceTransformtechnique issameas that obtained by standard differential method .

DCRESPONSEOFANR-CCIRCUIT(L.T.Method)

Similarly,

Let usdeterminethesolutionofthefirst orderdifferential equationgivenbyequationAwhich is for the DC response of a R-C Circuit under the zero initial condition i.e. voltage across capacitor is zero, V_c=0att= $0^-$and hence V_c=0at t= 0^+in thecircuit in figure A bytheproperty ofcapacitancenot allowingthevoltageacrossittochangeasswitchisclosedatt=0.



FigureLT1.2

V=Ri+ $\frac{1}{C}\int i\,dt$...LT1.5

TakingtheLaplaceTransformofbothsidesweget,

$\frac{V}{s}$=R I(s) + $\frac{1}{}[\frac{I(s)}{Cs}$+I (0)] LT1.6

$=> \frac{V}{s}$=R I(s) + $\frac{1}{}[\frac{I(s)}{Cs}]$ (I(0)=0 :zeroinitialcharge)

$=> \frac{V}{s}$ =I(s)[R+ $\frac{1}{Cs}$]=I(s)[$\frac{RCs+1}{Cs}$]

$\Rightarrow I(s) = \frac{V}{s}\left[\frac{Cs}{(RCs+1)}\right] = \frac{VC}{(RCs+1)}$.................................LT1.7

TakingtheLaplaceInverseTransformofbothsidesweget,

$\Rightarrow L^{-1}\{I(s)\} = i(t) = L^{-1}\{\frac{VC}{(RCs+1)}\}$

i(t)= $L^{-1}\{\frac{\frac{VC}{RC}}{\left[s+\frac{1}{RC}\right]}\}$(DividingthenumeratoranddenominatorbyRC)

putting $\propto = \frac{1}{RC}$ weget

i(t) = $L^{-1}\{\frac{V/R}{[s+\alpha]}\} = \frac{V}{R}e^{-\alpha t}$

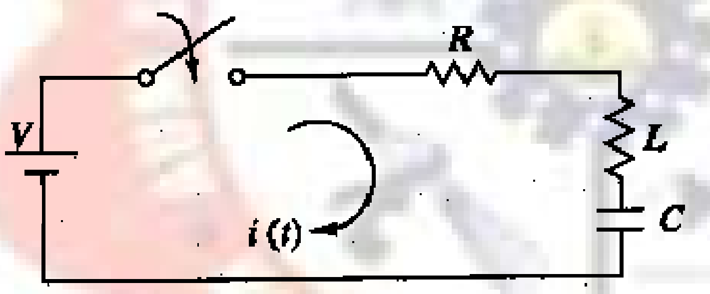
i(t)= $\frac{V}{R}e^{\frac{-t}{RC}}$(puttingbackthevalueof $\propto$)

i(t)= $I_o e^{\frac{-t}{RC}}$(where $I_o = \frac{V}{R}$)...................................LT1.8

i(t)= $I_o e^{\frac{-t}{\tau}}$) (where $\tau = Time\ constant$ = RC)

It canbeobserved that solutionfori(t)asobtainedbyLaplaceTransformtechniquein qis same as that obtained by standard differential method in d.

DCRESPONSEOFANR-L-CCIRCUIT(L.T.Method)



FigureLT1.3

Similarly,

Let us determinethesolution iofthefirst orderdifferential equationgiven byequationA which isfortheDCresponseofa R-L-CCircuit underthezeroinitial condition i.e.theswitchsisclosed at t=0.at t=0-,i.e. just before closing the switch s , the current in the inductor is zero. Since the inductor does not allow sudden changes in currents, at t=o+ just after the switch is closed,the current remains zero.alsothevoltageacrosscapacitor iszeroi.e. V_c=0att=0^-andhence V_c=0 att=0^+inthecircuitinfigurebythepropertyofcapacitancenot allowingthevoltageacross it $V_c$to suddenly change as switch is closed at t=0.

V=Ri +L.... $\frac{di}{dt}$ + $\frac{1}{C}\int i\,dt$.. LT1.9

TakingtheLaplaceTransformofbothsidesweget,

$\frac{V}{s}$=R I(s) ++ L [sI(s) –I(0)]+$\frac{1}{C}$[$\frac{I(s)}{s}$+I (0)] LT1.10

=> $\frac{V}{s}$=R I(s) + L[s I(s)] + $\frac{1}{C}$[$\frac{I(s)}{s}$]   ($I(0) = 0$: zero initial current &I(0)=0:zeroinitial charge)

=> $\frac{V}{s}$= I(s)[R +Ls + $\frac{1}{Cs}$] = I(s)[$\frac{LCs^2+RCs+1}{Cs}$]

=>I(s)=$\frac{V}{s}$[$\frac{Cs}{(LCs^2+RCs+1)}$] =$\frac{VC}{(LCs^2+RCs+1)}$ LT1.11

TakingtheLaplaceInverseTransformofbothsidesweget,

=> L^{-1}{I(s)}=$i(t)$ = $L^{-1}\{\frac{VC}{(LCs^2+RCs+1)}\}$

i(t)=$L^{-1}\{\frac{\frac{VC}{LC}}{[s^2+s\frac{RC}{LC}+\frac{1}{LC}]}\}$(DividingthenumeratoranddenominatorbyLC)

i(t) =$L^{-1}\{\frac{\frac{V}{L}}{[s^2+s\frac{R}{L}+\frac{1}{LC}]}\}$

putting $\propto$ =$\frac{R}{2L}$ and ω= $\sqrt{\frac{1}{LC}}$weget

i(t) =$L^{-1}\{\frac{\frac{V}{L}}{[s^2+2\propto s+\omega^2]}\}$

Thedenominatorpolynomialbecomes=$[s^2+2\propto s+\omega^2]$

where, s_1 , s_2 = $\frac{-2\propto\pm\sqrt{4\propto^2-4\omega^2}}{2}$=$-\propto\pm\sqrt{\propto^2-\omega^2}$=$-\propto\pm\beta$

where,$\propto$= $\frac{R}{2L}$;ω=$\sqrt{\frac{1}{LC}}$andβ=$\sqrt{\propto^2-\omega^2}$

BypartialFractionexpansion,ofI(s),

$$I(s)=\frac{A}{s-s_1}+\frac{B}{s-s_2}$$

$$A=(s-s_1)I(s)]s=s_1$$

$$=\frac{\frac{V}{L}}{(s_1-s_2)}$$

$$B=(s-s_2)I(s)]s=s_2$$

$$=\frac{\frac{V}{L}}{(s_2-s_1)}=-\frac{\frac{V}{L}}{(s_1-s_2)}$$

$$I(s)=\frac{\frac{V}{L}}{(s_1-s_2)}\left(\frac{1}{(s-s_1)}-\frac{1}{(s-s_2)}\right)$$

TakingtheInverseLaplaceTransform

$i(t)=A_1e^{s_1t}+A_2e^{s_2t}$

Where A_1 and A_2 are constants to be determined and s_1 and s_2 aren the roots of the equation.

Now depending upon the values of s_1 and s_2, we have three cases of the response. CASE I :

When the roots are Real and Unequal, it gives an over-damped response.

$\frac{R}{2L} > \sqrt{\frac{1}{LC}}$ or $\alpha > \omega$; In this case, the solution is given by i(t) =

$e^{-\alpha t}(A_1e^{\beta t}+A_2e^{-\beta t})$ ....LT 1.12

or $i(t)=A_1e^{s_1t}+A_2e^{s_2t}$ for $t>0$

CASE II: When the roots are Real and Equal, it gives an Critically-damped response.

$\frac{R}{2L}=\sqrt{\frac{1}{LC}}$ or $\alpha=\omega$; In this case, the solution is given by or

$i(t)=e^{-\alpha t}(A_1+A_2t)$ for $t>0$... LT1.13

CASE III: When the roots are Complex Conjugate, it gives an under-damped response.

$\frac{R}{2L} < \sqrt{\frac{1}{LC}}$ or $\alpha < \omega$; In this case, the solution is given by i(t) =

$A_1e^{s_1t}+A_2e^{s_2t}$ for $t > 0$

where, $s_1, s_2 = \frac{-2\alpha \pm \sqrt{4\alpha^2-4\omega^2}}{2} = -\alpha \pm \sqrt{\alpha^2-\omega^2}$

Let $\sqrt{\alpha^2-\omega^2} = \sqrt{-1}\sqrt{\omega^2-\alpha^2} = j\omega_d$ where $j=\sqrt{-1}$ and $\omega_d=\sqrt{\omega^2-\alpha^2}$

Hence, $i(t)=e^{-\alpha t}(A_1e^{j\omega_d t}+A_2e^{-j\omega_d t})$

$$i(t)=e^{-\alpha t}\left[(A_1+A_2)\left\{\frac{e^{j\omega_d t}+e^{-j\omega_d t}}{2}\right\}+j(A_1-A_2)\left\{\frac{e^{j\omega_d t}-e^{-j\omega_d t}}{2j}\right\}\right]$$

$$i(t)=e^{-\alpha t}[(A_1+A_2)\cos\omega_d t+j(A_1-A_2)\sin\omega_d t]$$

$i(t)=e^{-\alpha t}(B_1\cos\omega_d t+B_2\sin\omega_d t)$....................................LT 1.14

,,xxxxxxxxxxxxxxxxx,,

TWOPORTNETWORKS

Generally, any network may be represented schematically by a rectangular box. A network may be used for representing either Source orLoad , or for a variety of purposes. A pair of terminals at whichasignalmayenterorleaveanetworkiscalledaport.Aportisdefinedasanypairof terminals into which energy is withdrawn ,or where the network variables may be measured .One such network having only one pair of terminals (1-1')is shown figure 1.1.

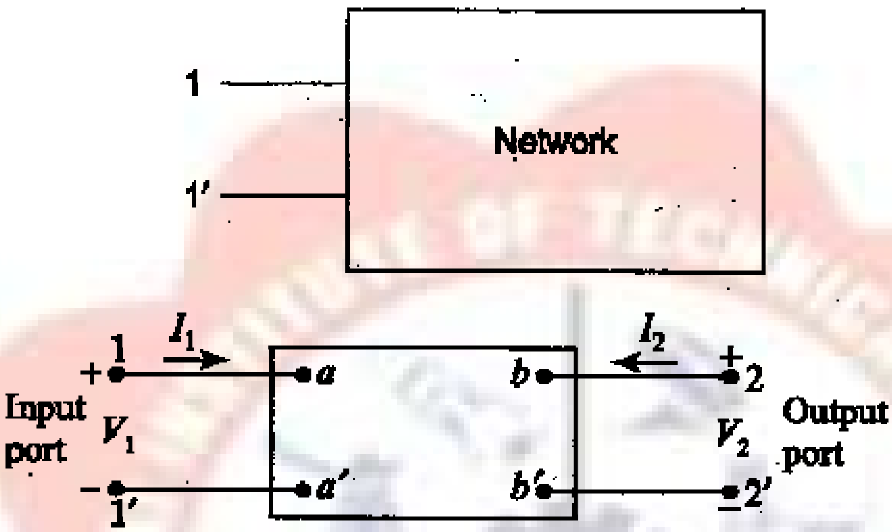


Figure1.1

A two-port network is simply a network a network inside a black box, and the network has only two pairsofaccessibleterminals;usuallyoneonepairsrepresentstheinputand theotherrepresentsthe output. Such a building block is very common in electronic systems, communication system, transmission and distribution system. fig 1.1 shows a two-port network,or two terminal pair network,in which the four terminals have been paired into ports 1-1' and 2-2'.The terminals 1-1' togetherconstituteaport.Similarly,theterminals2-2'constituteanother port.Twoportscontaining nosources in their branches are calledpassive ports ; among them are power transmissionlines and transformers. Two ports containing source in their branches are called active ports. A voltage and currentassigned to each of the two ports. The voltageand current at the input terminals are $V_1$and I_1; where as$V_2$and$I_2$are entering into the network are V_1, V_2,andI_1, I_2. Two of these are dependent variable, the other two are indepent variable. The number of possible combinations generatedbyfourvariable,takentwoattime,issix.Thus,therearesixpossiblesetsofequations describing a two-port network.

OPENCIRCUITIMPEDANCE(Z)PARAMETERS

Agenerallineartwo-portnetworkisshownbelowinfigure1.2.

Thezparametersofa two-portnetworkfor thepositivedirectionofvoltagesandcurrents maybe definedbyexpressingtheportvoltagesV_1andV_2intermsofthecurrents I_1andI_2.Here$V_1$and V_2aretwo dependentvariables and $I_1$and$I_2$are two independent variables.

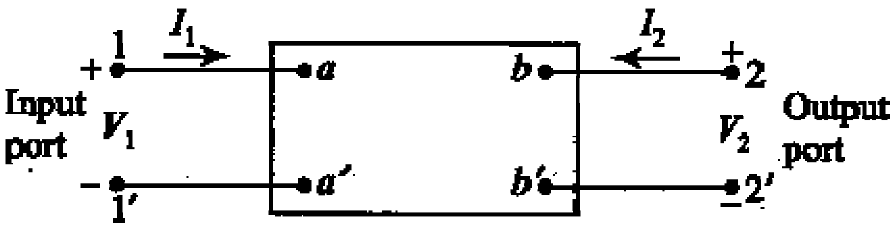


Figure1.2

Thevoltageatport1-1'istheresponseproducedbythetwocurrents $I_1$and I_2. thus

$V_1 = Z_{11} I_1 + Z_{12} I_2$.. 1.1

$V_2 = Z_{21} I_1 + Z_{22} I_2$.. 1.2

Z_{11}, Z_{12}, Z_{21} *and* Z_{22}arethenetworkfunctions,andarecalledimpedance(Z)parameters,andare defined by equations1.1 and 1.2 .

Theseparametersalsocanberepresentedby Matrices. We

may write the matrix equation [V] = [Z][I]

whereVisthecolumnmatrix=$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$ Z is a

square matrix =$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$

andwemaywrite$[I]$inthecolumnmatrix==$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$ Thus,

$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$

TheindividualZparametersforagivennetworkcanbedefinedbysettingeachofthe portcurrents equal tozero. suppose port2-2' is left open circuited, then I_2=0.

ThusZ_{11}=$\frac{V_1}{I_1}\Big|_{I_2} = 0$where

Z_{11} is the driving point impedance at port 1 − 1' with port 2 − 2' open circuited. It is called the open circuit input impedance.

similarly,

Z_{21}=$\frac{V_2}{I_1}\Big|_{I_2} = 0$

where
Z_{21} is the transfer impedance at port 1 − 1' with port 2 − 2' open circuited. It is called the open circuit forward transfer impedance

.

Supposeport1-1'isleftopencircuited,thenI_1=0.

Thus, $Z_{12}=\frac{V_1}{I_2}\Big|_{I_1=0}$

where

Z_{12} is the transfer impedance at port $2-2'$ with port $1-1'$ open circuited. It is called the open circuit reverse transfer impedance.

similarly,

$Z_{22}=\frac{V_2}{I_2}\Big|_{I_1=0}$

where

Z_{22} is the open circuit driving point impedance at port $2-2'$ with port $1-1'$ open circuited. It is also called the open circuit output impedance

.Theequivalentcircuitofthetwo-portnetworksgovernedbytheequations 1.1and 1.2,i.e.open circuit impedance parameters as shown below in fig 1.3.

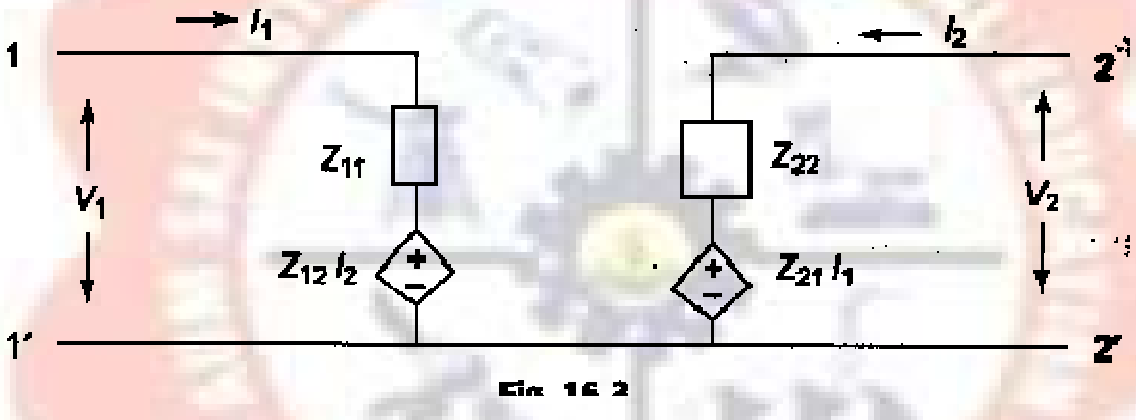


Figure1.3

Ifthenetworkunderstudyisreciprocalorbilateral,theninaccordancewiththereciprocityprinciple $I_2 = 0$

$\frac{V_2}{I_1}\Big| = \frac{V_1}{I_2}\Big|_{I_1=0}$

or

$Z_{21} = Z_{12}$

It is observed that all the parameters have the dimensions of impedance. Moreover, individual parametersarespecifiedonlywhenthecurrentinone ofthe portsiszero. Thiscorresponds toone of the ports being open circuited from which the Z parameters also derive the name open circuit impedance parameters.

Problem1.1

FindtheZparametersforthe circuitshowninFigure1.4

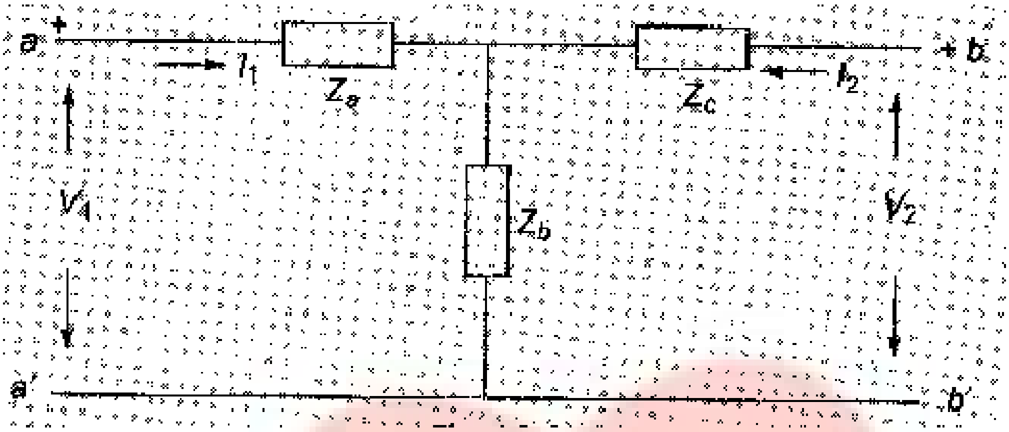


Figure1.4

SolutionThecircuitintheproblemisaTnetwork.FromEqs16.1and16.2wehave

$V_1 = Z_{11} I_1 + Z_{12} I_2$ and $V_2 = Z_{21} I_1 + Z_{22} I_2$

When port b-b' is open circuited,

$Z_{11} = \frac{V_1}{I_1}$

Where $V_1 = I_1(Z_a + Z_b)$

$\therefore Z_{11} = (Z_a + Z_b)$

$Z_{21} = \frac{V_2}{I_1}\Big|\ I_2 = 0$

Where $V_2 = I_1 Z_b$ $\therefore Z_{21} = Z_b$

When port a-a' is open circuited, I_1=0

$Z_{22} = \frac{V_2}{I_2}\Big| I_1 = 0$

where $V_2 = I_2(Z_b + Z_c)$

$Z_{22} = (Z_b + Z_c)$

$Z_{12} = \frac{V_1}{I_2}\Big| I_1 = 0$

where $V_1 = I_2 Z_b$ and $Z_{12} = Z_b$

Itcanbeobservedthat $Z_{12} = Z_{21}$, sothenetworkisabilateral networkwhichsatisfiesthe principle of reciprocity.

SHORT-CIRCUITADMITTANCE(Y)PARAMETERS

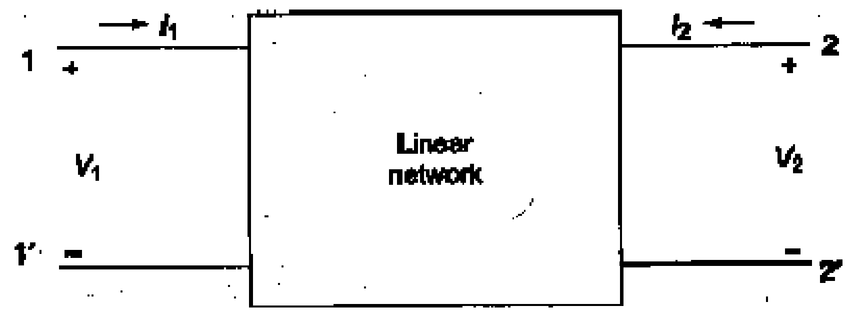


Figure 1.5

Ageneraltwo-portnetworkwhichisconsideredinSection16.2isshown inFig16.5TheY parameters of a two- port for the positive directions of voltages and currents may be defined by expressingtheportcurrentsI_1andI_2intermsofthevoltages V_1andV_2. HereI_1, I_2aredependent variables and$V_1$and$V_2$are independentvariables. I_1may be considered tobe the superpositionof two components, one caused by$V_1$and the other byV_2.

Thus,

$$I_1 = Y_{11}V_1 + Y_{12}V_2 \quad \ldots\ldots 1.3$$

Similarly, $$I_2 = Y_{21}V_1 + Y_{22}V_2 \quad \ldots\ldots 1.4$$

Y_{11}, Y_{12}, Y_{21}andY_{22}arethenetworknetworkfunctionsandarealsocalledtheadmittance (Y)parameters.TheyaredefinedbyEqs16.3and16.4. Theseparameterscanberepresented by matrices as follows

[I]=[Y][V]

whereI=$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$;Y=$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$andV= $\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$ Thus,

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}=\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

TheindividualYparametersforagivennetworkcanbedefinedbysettingeachportvoltagetozero. IfweletV_2bezerobyshortcircuitingport2-2'then

$$Y_{11}=\frac{I_1}{V_1}\Big|\, V_2=0$$

Y_{11}isthedrivingpointadmittanceatport1-1', withport 2-2' short circuited.Itisalsocalledthe shortcircuitinputadmittance.

$$Y_{21}= \frac{I_2}{V_1}\Big|\, V_2=0$$

Y_{21}isthetransferadmittanceatport1-1', withport 2-2' shortcircuited.Itisalsocalledtheshort circuitedforwardtransferadmittance.IfweletV_1*b*ezerobyshortcircuitingport1-1',then

$Y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0}$

Y_{12} is the transfer admittance atport2-2’, withport1-1’shortcircuited. Itis alsocalledthe short circuitedreversetransferadmittance.

$Y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0}$

Y_{22} istheshortcircuitdriving pointadmittanceat port 2-2’, withport1-1’ short circuited. Itisalso calledtheshortcircuited outputadmittance.Theequivalentcircuitofthenetworkgovernedby equation 1.3 & 1.4 is shown in figure 1.6.

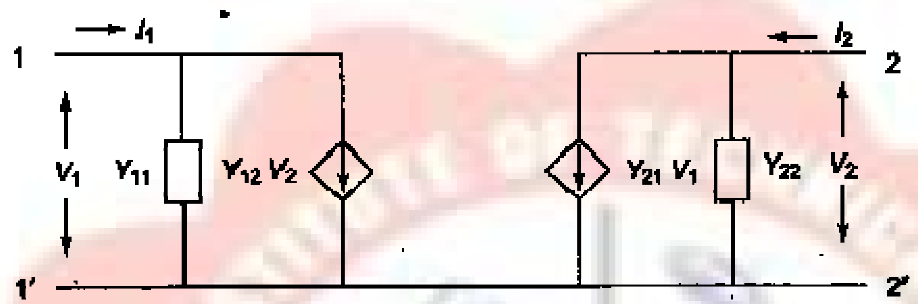


Figure 1.6

Ifthenetworkunderstudyisreciprocalorbilateral,theninaccordancewiththereciprocityprinciple $V_1 = 0$

$\frac{I_1}{V_2}\Big| \quad = \frac{I_2}{V_1}\Big|_{V_2=0}$

or

$Y_{12} = Y_{21}$

It is observed that all the parameters have the dimensions of admittance. Moreover, individual parametersarespecifiedonlywhenthe voltageinoneofthe portsiszero. Thiscorresponds toone of the ports being short circuited from which the Y parameters also derive the name short circuit admittance parameters.

Problem1.2FindtheY-parametersforthenetworkshowninFig.1.7

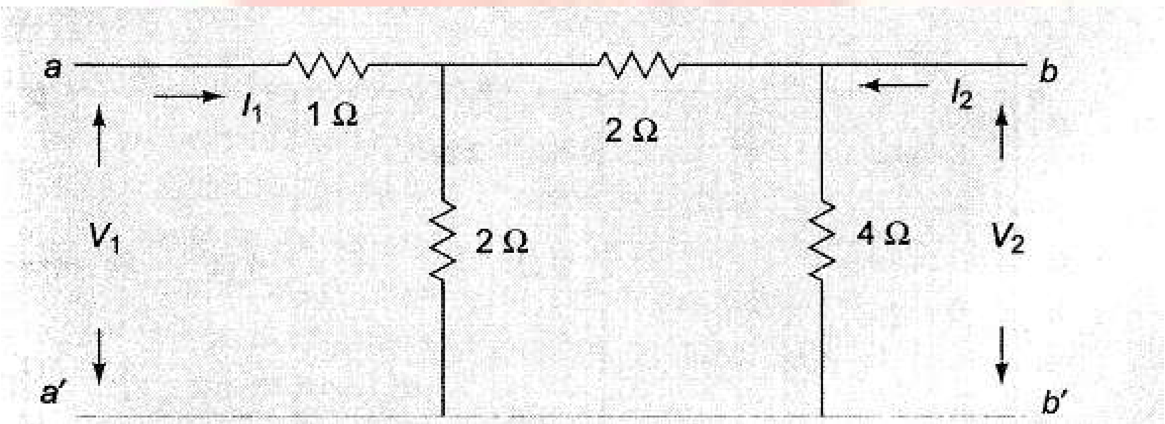


Fig1.7

Solution:

$Y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0}$

When b-b' is short circuited, $V_2=0$ and the network looks as shown in Fig.1.8(a)

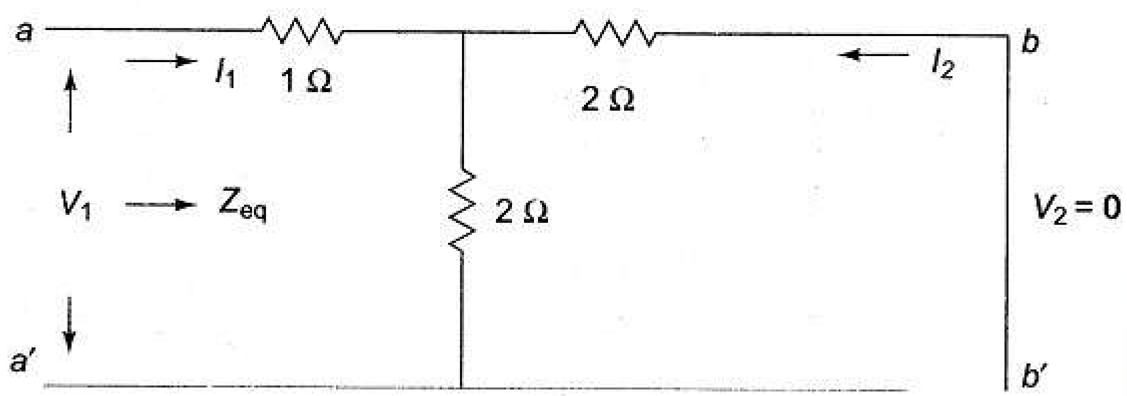


Fig.1.8(a)

$V_1 = I_1 Z_{eq}$

$Z_{eq} = 2\ \Omega$

So, $V_1 = I_1\, 2$

$Y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0} = \frac{I_1}{V_1} = \frac{1}{2}$

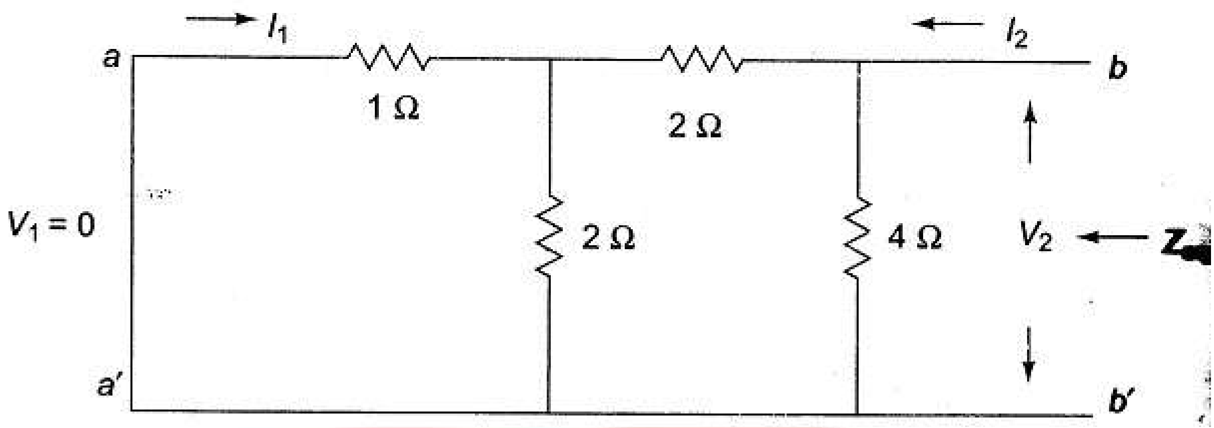
$Y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0}$

When b-b' is short circuited, $-I_2 = I_1 \times \frac{2}{4} = \frac{I_1}{2}$

so, $-I_2 = \frac{V_1}{4}$

and $Y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0} = -\frac{1}{4}$

similarly, when port a-a' is short circuited, $V_1=0$ and the network looks as shown in Fig. 1.8(b)



$Y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0}$

$V_2 = I_2 Z_{eq}$ where Z_{eq} istheequivalentimpedanceasviewedfromb-b′. $Z_{eq} =$

$\frac{8}{5}\,\Omega$

$V_2 = I_2 \times \frac{8}{5}$

$Y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0} = \frac{5}{8}$

$Y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0}$

witha-a′isshortcircuited, $-I_1 = \frac{2}{5} I_2$ Since ,

$I_2 = 5\,\frac{V_2}{8}$

$-I_1 = \frac{2}{5} \times 5\,\frac{V_2}{8} = \frac{V_2}{4}$

So, $Y_{12} = \frac{I_1}{V_2} = \frac{1}{4}$

Thedescribingequationsintermsoftyeadmittanceparametersare

$$I_1 = \frac{1}{2}V_1 + \frac{1}{4}V_2$$

$$I_2 = -\frac{1}{4}V_1 + \frac{5}{8}V_2$$

Transmission(ABCD)parameters

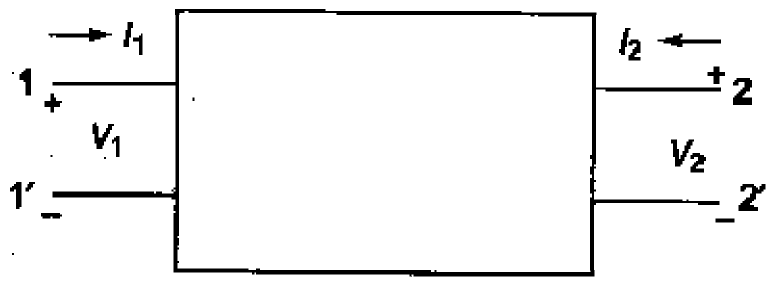


Figure1.9

Transmission parameters or ABCD parameters are widely used in transmission line theory and cascadednetworks.Indescribingthetransmissionparameters,theinputvariablesV_1andI_1atport 1-1', usually calledthesending end areexpressed intermsof the output variables V_2andI_2at port 2-2', called, the receiving end.The transmission parameters provide a direct relationship between input and output.Transmissionpatameters are also called general circuit parameters, or chain nparameters. They are defined by

$V_1 = AV_2 - BI_2$.. 1.5

$I_1 = CV_2 - DI_2$..1.6

ThenegativesignisusedwithI_2,andnotfortheparameterBandD.Boththeportcurrents $I_1$and- $I_2$are directed to the right, i.e. with a negative sign in equation a and b the currents at port 2-2' which leaves the port is designated as positive.The parameters A,B,C and d are called Transmission parameters. In the matrix form, equation a and b are expressed as ,

$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix}=\begin{bmatrix} A & D \\ C & D \end{bmatrix}\begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$

Thematrix $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$is called Transmission Matrix.

Foragivennetwork,theseparameterscanbedeterminedasfollows.Withport2-2'opencircuited i.e.I_2=0;applyingavoltageV_1attheport1-1',usingequa,wehave

A =$\frac{V_1}{V_2}\Big| I_2 = 0$andC =$\frac{I_1}{V_2}\Big| I_2 = 0$

hence, $\frac{1}{A}$=$\frac{V_2}{V_1}\Big| I_2 = 0$=$g_{21}$| I_2=0

1/Aiscalledtheopencircuitvoltagegainadimensionlessparameter.And $\frac{1}{C}$=$\frac{V_2}{I_1}\Big| I_2 = 0$=$Z_{21}$| I_2 =0is called open circuit transfer impedance. with port 2-2' short circuited, i.e. V_2 =0,applying voltageV_1at port 1-1' from equn . b we have

-B= $\frac{V_1}{I_2}\Big| V_2 = 0$ and-D= $\frac{I_1}{I_2}\Big| V_2 = 0$

$-\frac{1}{B} = \frac{I_2}{V_1}\Big| V_2 = 0$ $= Y_{21} | V_2$ =0iscalledshortcircuittransferadmittance

and,

$-\frac{1}{D} = \frac{I_2}{I_1}\Big| V_2 = 0$ $= \alpha_{21} | V_2$ =0iscalledshortcircuitcurrentgainadimensionlessparameter.

Problem1.3

Findthetransmissionorgeneralcircuitparametersforthecircuitshowninfig.1.10

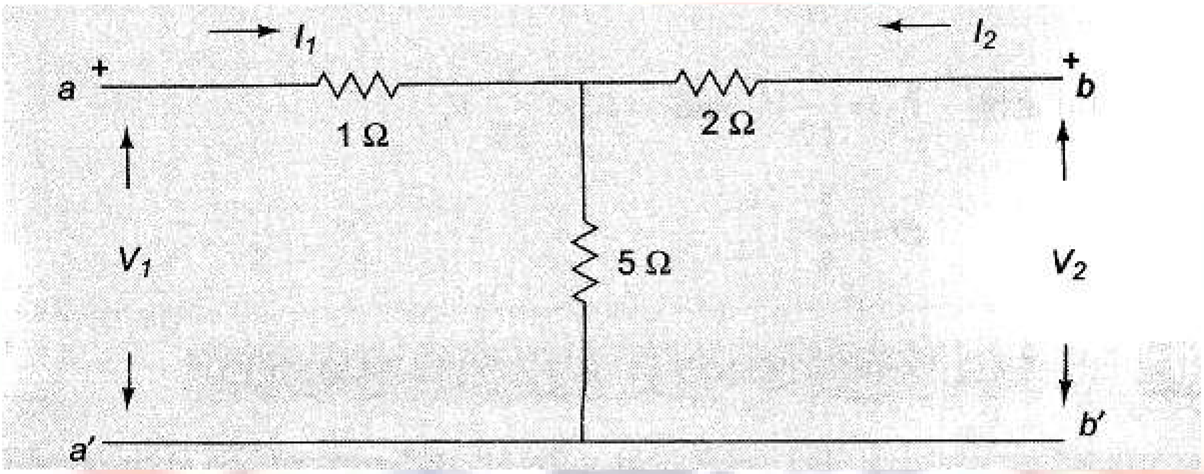


Fig.1.10

Solution:FromEquations1.5and1.6,wehave

$V_1 = AV_2 - BI_2$

$I_1 = CV_2 - DI_2$

whenb-b’isopencircuitedi.e.I_2=0,wehave A =

$\frac{V_1}{V_2}\Big| I_2 = 0$

whereV_1=$6\ I_1$and V_2=$5\ I_1$andhence,A=$\frac{6}{5}$ andC =

$\frac{I_1}{V_2}\Big| I_2 = 0$= $\frac{1}{5}$

whenb-b’isshortcircuitedi.e.V_2=0,wehave B = -

$\frac{V_1}{I_2}\Big| V_2 = 0$andD =-$\frac{I_1}{I_2}\Big| V_2 = 0$

Inthecircuit,-I_2= $\frac{5}{17} V_1$ andso,B= $\frac{17}{5}\,\Omega$

similarly, $I_1 = \frac{7}{17} V_1$ and $-I_2 = \frac{5}{17} V_1$ and

hence D = $\frac{7}{5}$

Hybridparameters

Hybridparametersorh-parametersfindextensiveuseintransistorcircuits.Theyarewell suitedto transistor circuits as these parameters can be most conveniently measured. The hybrid matrices describeatwo-portnetwork,whenthevoltageofone portand thecurrentof otherportare taken as the independent variables. Consider the network in figure 1.11.

Ifthevoltageatport1-1'andcurrentatport2-2'are takenasdependentvariables,wecan expressthemintermsof I_1 and V_2.

$V_1 = h_{11} I_1 + h_{12} V_2$ 1.7

$I_2 = h_{21} I_1 + h_{22} V_2$ 1.8

Thecoefficientintheabovetermsarecalledhybridparameters.Inmatrixnotation $\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$

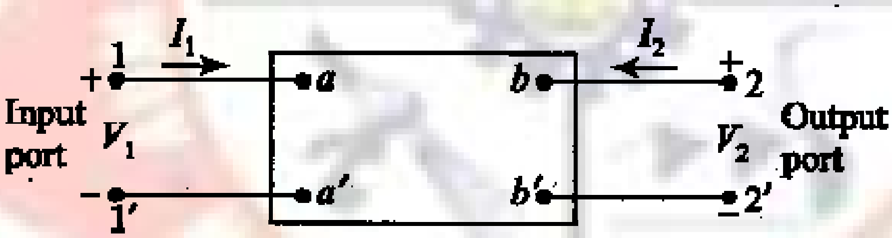


Figure1.11

fromequationaandbtheindividualhparametersmaybedefinedbyletting $I_1 = 0$ and V_2=0. when V_2= 0,theport 2-2' is short circuited.

Then $h_{11} = \frac{V_1}{I_1}\Big|_{V_2=0}$ =shortcircuitinputimpedance. $h_{21} = \frac{I_2}{I_1}\Big|_{V_2=0}$ = short circuit forward current gain Similarly,

by letting port 1-1' open, $I_1 = 0$

$h_{12} = \frac{V_1}{V_2}\Big|_{I_1=0}$ =opencircuitreversevoltagegain

$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$ =opencircuitedoutputadmittance

Since h-parameters represent dimensionally an impedance, an admittance,a voltage gain and a currentgain,theyarecalledhybridparameters.Anequivalentcircuitofatwo-portnetworkin terms of hybrid parameters is shown below.

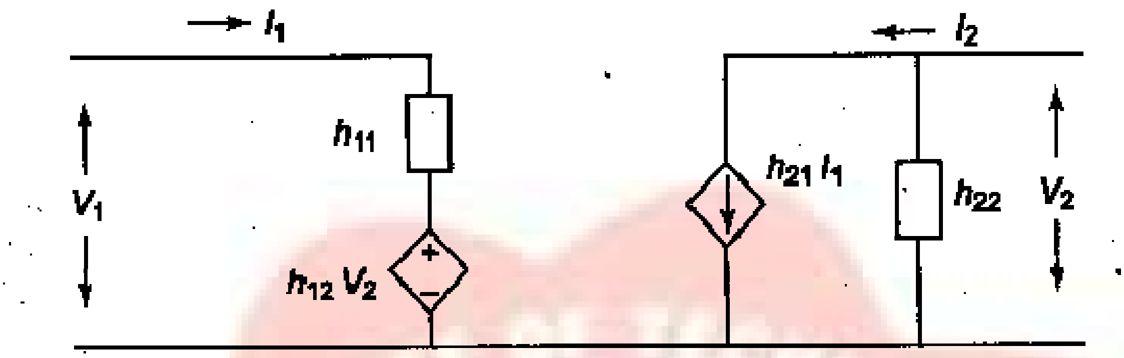


Figure1.12

Problem1.4

Findtheh-parameters ofthenetworkshowninFig1.13.

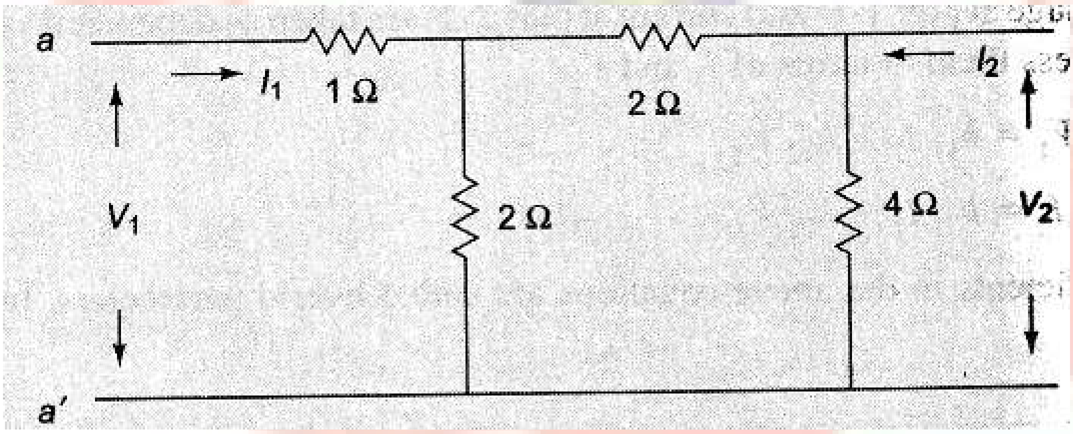


Fig.1.13

Solution :

Fromequations1.7and1.8, wehave

$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}$; $h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$; $h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$; $h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$

Ifportb-b′isshortcircuited,V_2=0andthenetworklooksasshowninFig.1.14(a)

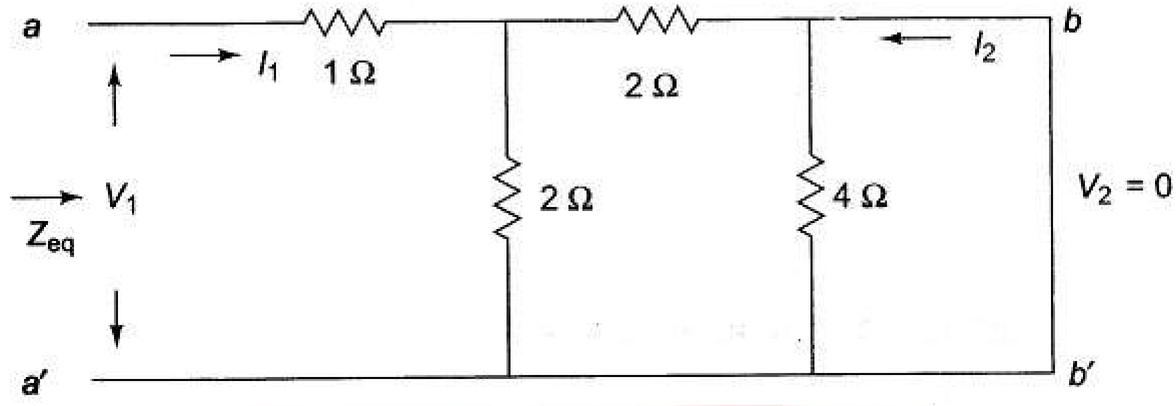


Fig.1.14(a)

$h_{21}= \frac{I_2}{I_1}\Big|_{V_2=0}$; $V_1 = I_1 Z_{eq}$

Z_{eq} istheequivalentimpedanceasviewedfromporta-a'is2Ω

so, $V_1=I_1 2$V h_{11}

$=\frac{V_1}{I_1}=2\Omega$

$h_{21}= \frac{I_2}{I_1}\Big|_{V_2=0}$ when $V_2=0$; $-I_2=\frac{I_1}{2}$ andhence $h_{21}=-\frac{1}{2}$

Ifporta-a'isopencircuited, $I_1=0$andthenetworklooksasshowninFig.1.14(b)then

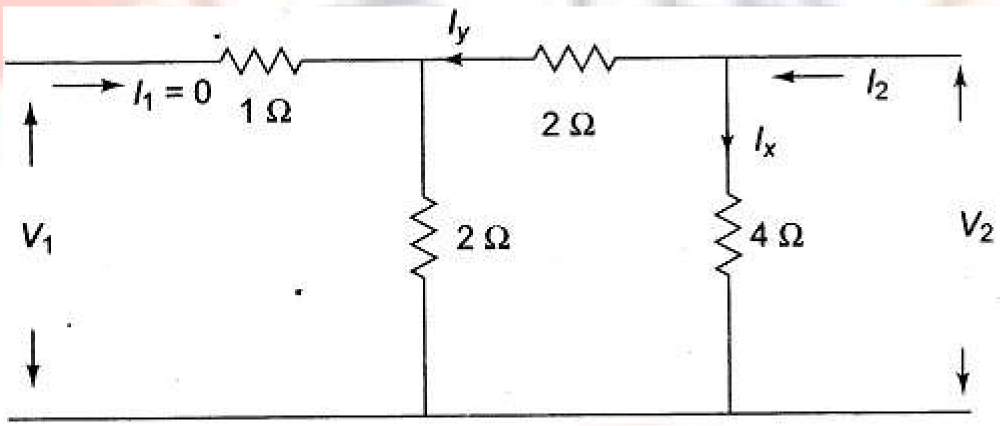


Fig.1.14(b)

$h_{12}= \frac{V_1}{V_2}\Big|_{I_1=0}$ and $V_1= I_y 2$; $I_y=\frac{I_2}{2}$ $V_2=$

$I_x 4$; $I_x=$ $\frac{I_2}{2}$

$h_{12}= \frac{V_1}{V_2}\Big|_{I_1=0}$ = and $h_{22}= \frac{I_2}{V_2}\Big|_{I_1=0}$ = $\frac{1}{2}$

INTERRELATIONSHIPSOFDIFFERENTPARAMETERS

ExpressionofzparametersintermsofYparametersandvice-versa

From equations 1.1,1.2,1.3 &1.4 , it is easy to derive the relation between the open circuit impedanceparametersandtheshortcircuitadmittanceparametersby meansoftwo matrix equationsof the respective parameters. By solving equation a and b for I_1andI_2, we get

$I_1= \begin{bmatrix} V_1 & Z_{12} \\ V_2 & Z_{22} \end{bmatrix}/\Delta_z$;and$I_2=\begin{bmatrix} Z_{11} & V_1 \\ Z_{21} & V_2 \end{bmatrix}/\Delta_z$

where Δ_z is the determinant of Z matrix

$\Delta_z=\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$

$I_1=\frac{Z_{22}}{\Delta_z}V_1-\frac{Z_{12}}{\Delta_z}V_2$..1.9

$I_2=-\frac{Z_{21}}{\Delta_z}V_1+\frac{Z_{11}}{\Delta_z}V_2$..1.10

comparingequations1.9and1.10withequations1.3and1.4wehave

$Y_{11} = \frac{Z_{22}}{\Delta_z}$;$Y_{12}= -\frac{Z_{12}}{\Delta_z}$

$Y_{21}= -\frac{Z_{21}}{\Delta_z}$;$Y_{22}=\frac{Z_{11}}{\Delta_z}$

Inasimilarmanner,thezparametersmaybeexpressedintermsofthe admittanceparametersby solving equations1.3and 1.4 for V_1andV_2

$V_1= \begin{bmatrix} I_1 & Y_{12} \\ I_2 & Y_{22} \end{bmatrix}/\Delta_y$;and$V_2=\begin{bmatrix} Y_{11} & I_1 \\ Y_{21} & I_2 \end{bmatrix}/\Delta_z$

where Δ_y is the determinant of Y matrix

$\Delta_y= \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$

$V_1=\frac{Y_{22}}{\Delta_y}I_1-\frac{Y_{12}}{\Delta_y}I_2$..1.11 $V_2=-\frac{Y_{21}}{\Delta_y}$

$I_1+\frac{Y_{11}}{\Delta_y}I_2$..1.12

comparingequations1.11and1.12withequations1.1and1.2wehave

$Z_{11} = \frac{Y_{22}}{\Delta_y}$; $Z_{12} = -\frac{Y_{12}}{\Delta_y}$

$Z_{21} = -\frac{Y_{21}}{\Delta_y}$; $Z_{22} = \frac{Y_{11}}{\Delta_y}$

GeneralCircuitParametersor ABCD Parameters inTermsofZparametersand Y Parameters

Weknow that

$V_1 = AV_2 - BI_2$; $V_1 = Z_{11}I_1 + Z_{12}I_2$; $I_1 = Y_{11}V_1 + Y_{12}V_2$

$I_1 = CV_2 - DI_2$; $V_2 = Z_{21}I_1 + Z_{22}I_2$; $I_2 = Y_{21}V_1 + Y_{22}V_2$

A $= \frac{V_1}{V_2}\big| I_2 = 0$; C $= \frac{I_1}{V_2}\big| I_2 = 0$; B $= -\frac{V_1}{I_2}\big| V_2 = 0$; D $= -\frac{I_1}{I_2}\big| V_2 = 0$

SubstitutingtheconditionI_2=0inequations1.1and1.2weget A = $\frac{V_1}{V_2}\big|$

$I_2 = 0 = \frac{Z_{11}}{Z_{21}}$

SubstitutingtheconditionI_2=0inequations1.4weget,

A $= \frac{V_1}{V_2}\big| I_2 = 0 = \frac{Y_{22}}{Y_{21}}$

SubstitutingtheconditionI_2=0inequations1.2weget C =

$\frac{I_1}{V_2}\big| I_2 = 0 = \frac{1}{Z_{21}}$

Substitutingthecondition I_2=0inequation1.3and1.4andsolvingfor V_2gives$-I_1 \frac{Y_{21}}{\Delta_y}$Where$\Delta_y$is

the determinant of the admittance matrix

$\frac{I_1}{V_2}\big| I_2 = 0 = \frac{-\Delta_y}{Y_{21}}$=C

Substitutingthecondition V_2=0inequations1.4,weget

$\frac{V_1}{I_2}\big| V_2 = 0 = -\frac{1}{Y_{21}}$ = B

Substitutingthecondition V_2=0inequation1.1and1.2andsolvingfor I_2gives$-V_1 \frac{Z_{21}}{\Delta_z}$Where$\Delta_z$is

the determinant of theimpedance matrix

$$-\frac{V_1}{I_2}\bigg|_{V_2=0} = \frac{\Delta Z}{Z_{21}} = B$$

Substitutingthecondition V_2=0inequation1.2weget,

$$\frac{-I_1}{I_2}\bigg|_{V_2=0} = \frac{Z_{22}}{Z_{21}} = D$$

SubstitutingtheconditionV_2=0inequations1.3and1.4 we get

$$\frac{-I_1}{I_2}\bigg|_{V_2=0} = \frac{-Y_{22}}{Y_{21}} = D$$

T and π representation

A two-port network with any number of elementsmay be converted into a two-port three-element network. Thus, a two-port network may be represented by an equivalent T-network,i.e.threeimpedances areconnectedtogetherintheformofa Tasshowninfigure 1.15.

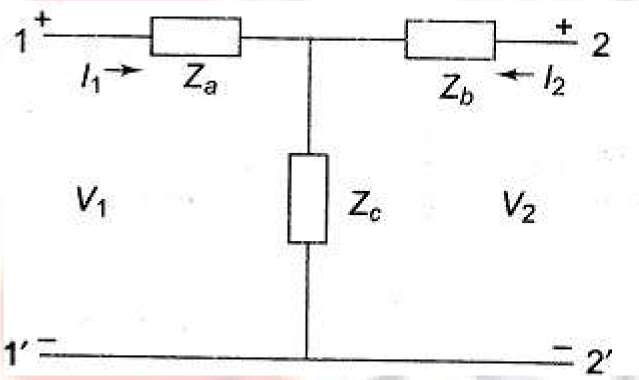


Figure1.15

ItispossibletoexpresstheelementsoftheT-networkintermofZ parameters,orABCD parametersasexplainedbelow.

Zparametersofthe network

$$Z_{11} = \frac{V_1}{I_1}\bigg|_{I_2=0} = Z_a + Z_c$$

$$Z_{21} = \frac{V_2}{I_1}\bigg|_{I_2=0} = Z_c$$

$$Z_{22}= \frac{V_2}{I_2}\Big| I_1 =0 \quad =Z_b + Z_c$$

$$Z_{12}= \frac{V_1}{I_2}\Big| I_1 =0 \quad = Z_c$$

Fromtheaboverelations,itisclearthat

$Z_a = Z_{11}- Z_{21}$

$Z_b = Z_{22}- Z_{12}$

$Z_c = Z_{12}\text{-}Z_{21}$

ABCDparametersofthe network

$A=\frac{V_1}{V_2}\Big| I_2=0 \quad = \frac{Z_a+Z_c}{Z_c}$

$B=\frac{-V_1}{I_2}\Big|V_2=0$

When2-2'isshortcircuited

$-I_2= \frac{V_1 Z_C}{Z_b Z_c+Z_a(Z_b+Z_c)}$

$B =(Z_a + Z_b)+\frac{Z_a Z_b}{Z_c}$

$C =\frac{I_1}{V_2}\Big| I_2=0 \quad = \frac{1}{Z_c}$

$D=\frac{-I_1}{I_2}\Big|V_2=0$

When2-2'isshortcircuited

$-I_2=I_1\frac{Z_c}{Z_b+Z_c}D$

$=\frac{Z_b+Z_c}{Z_c}$

Fromtheaboverelationswecanobtain Z_a

$= \frac{A-1}{C}; Z_b=\frac{D-1}{C}; Z_c= \frac{1}{C}$

Problem:1.6

The Z parameters of a Two-port network are $Z_{11} = 10\Omega, Z_{12} = 15\Omega, Z_{12} = Z_{21} = 5\Omega$.

Find the equivalent T network and ABCD Parameters.

Solution :

The equivalent T network is shown in Figure 1.16 where Z_a

$= Z_{11} - Z_{21} = 5\Omega$

$Z_b = Z_{22} - Z_{12} = 10\Omega$

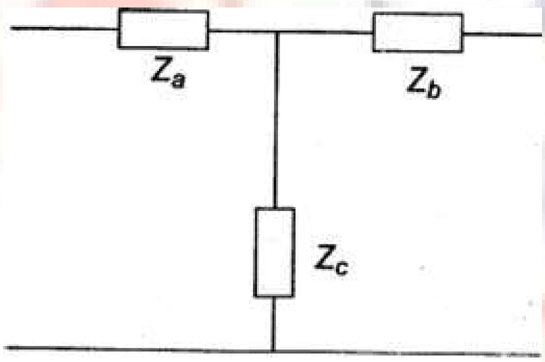
and $Z_c = 5\Omega$

The ABCD parameters of the network are A =

$\frac{Z_a}{Z_c} + 1 = 2$; B $= (Z_a + Z_b) + \frac{Z_a Z_b}{Z_c} = 25\ \Omega$

$C = \frac{1}{Z_c} = 0.02; D = 1 + \frac{Z_b}{Z_c} = 3$

In a similar way a two-port network may be represented by an equivalent π - network, i.e. three impedances or admittances are connected together in the form of π as shown in Fig 1.17.



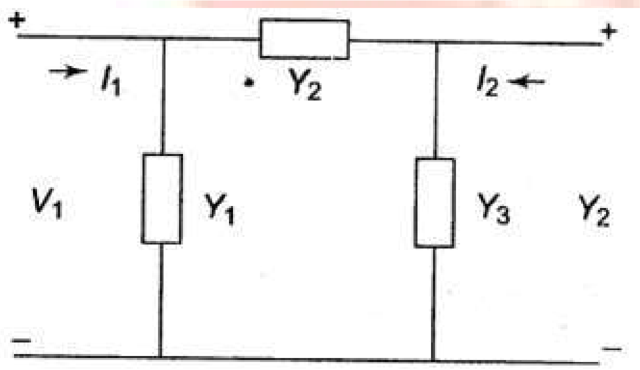


Fig.1.16 Fig.1.17

Itispossibletoexpresstheelementsoftheπ-networkintermsofYparametersor ABCD parametersasexplainedbelow.

Y-parametersofthenetwork

$Y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0} = Y_1 + Y_2$

$Y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0} = -Y_2$

$Y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0} = Y_3 + Y_2$

$Y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0} = -Y_2$

Fromtheaboverelations,itisclearthat $Y_1 =$

$Y_{11} + Y_{21}$

$Y_2 = -Y_{12}$

$Y_3 = Y_{22} + Y_{21}$

WritingABCDparametersintermsofYparametersyieldsthefollowingresults.

$A = \frac{-Y_{22}}{Y_{21}} = \frac{Y_3 + Y_2}{Y_2}$

$B = \frac{-1}{Y_{21}} = \frac{1}{Y_2}$

$C = \frac{-\Delta Y}{Y_{21}} = Y_1 + Y_3 + \frac{Y_1 Y_3}{Y_2}$

$D = \frac{-Y_{11}}{Y_{21}} = \frac{Y_1 + Y_2}{Y_2}$

fromtheaboveresults,weobtain

$Y_1 = \frac{D-1}{B}$; $Y_2 = \frac{1}{B}$; $Y_3 =$

$\frac{A-1}{B}$

....xxxxxxxxxxxxxxxxxxxx..

CLASSIFICATIONOFFILTERS

Afilterisareactivenetworkthatfreely passesthedesiredbandoffrequencieswhilealmost totally suppressing all other bands. A filter is constructed from purely reactive elements, for otherwise the attenuation would never becomeszero i n the pass band of thefilter network.

Filters differ from simple resonant circuit in providing a substantially constant transmission over the band which they accept; this band may lie between any limits depending on the design. Ideally, filters should produce no attenuation in the desired band, called the transmissionbandorpassband,andshould providetotal orinfiniteattenuationatallother frequencies, called attenuation band or stop band. The frequency which separates the transmissionbandandtheattenuationbandisdefinedasthecut-offfrequency ofthewave filters, and is designated by *fc*

Filter networks are widely used in communication systems to separate various voice channels in carrier frequency telephone circuits. Filters also find applications in instrumentation, telemetering equipment etc. where it is necessary to transmit or attenuate a limited range of frequencies. A filter may, in principle, have any number of pass bands separated by attenuation bands.However,theyareclassifiedintofourcommontypes,viz.lowpass,highpass,bandpassand band elimination.

Decibelandneper

The attenuation of a wave filter can be expressed in decibels or nepers.Neper is defined as the naturallogarithmoftheratioof inputvoltage(or current)to the outputvoltage(orcurrent),provide that the network is properly terminated in its characteristic impedance Z_0.

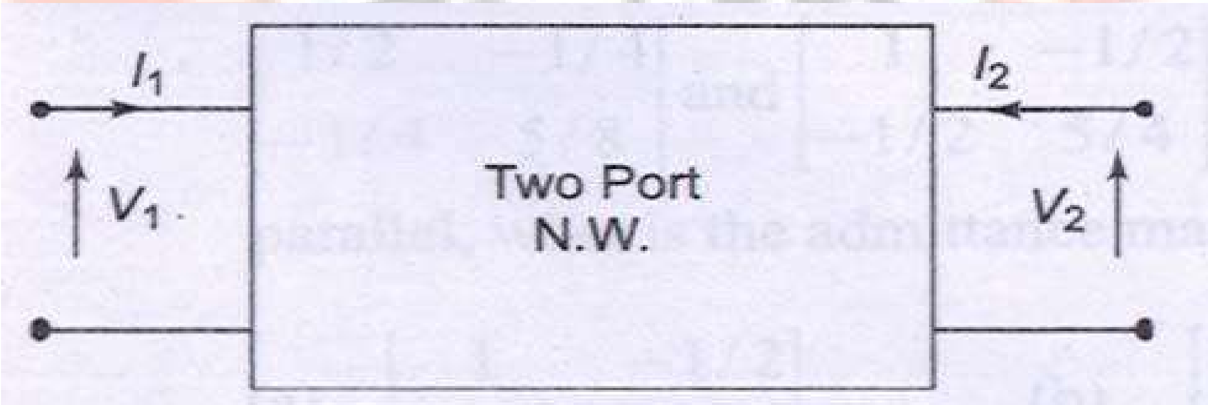


Fig.9.1(a)

From fig. 9.1 (a) the number of nepers, N= $log_e\,[V_1/V_2]$or $loge\,[I_1/I_2]$. A neper can also be expressed in terms of input power,$P_1$and the output power P_2as N=1/2 $log_e P_1/P_2$. A decibel is definedastentimesthecommonlogarithmsoftheratiooftheinputpowertotheoutputpower.

DecibelD=10$log_{10}P_1/P_2$

Thedecibelcanbeexpressedintermsof theratioof inputvoltage(orcurrent) andthe output voltage (or current.)

D=20log$_{10}$[V_1/V_2]=20log$_{10}$[I_1/I_2]

*Onedecibelisequalto0.115 N.

LowPassFilter

Bydefinitionalowpass(LP)filterisonewhichpasseswithoutattenuationallfrequencies up to the cut-off frequency f_c, and attenuates all otherfrequenciesgreaterthanf_c.The attenuation characteristic of an ideal LP filter is shown in fig.9.1(b).This transmits currents of all frequencies from zero up to the cut-off frequency. The band is called pass band or transmission band.Thus,the pass bandfortheLP filter is thefrequencyrange0 to f_c.Thefrequencyrange overwhichtransmissiondoesnottakeplaceiscalledthestopband orattenuationband. Thestop band for a LP filter is the frequency range above f_c.

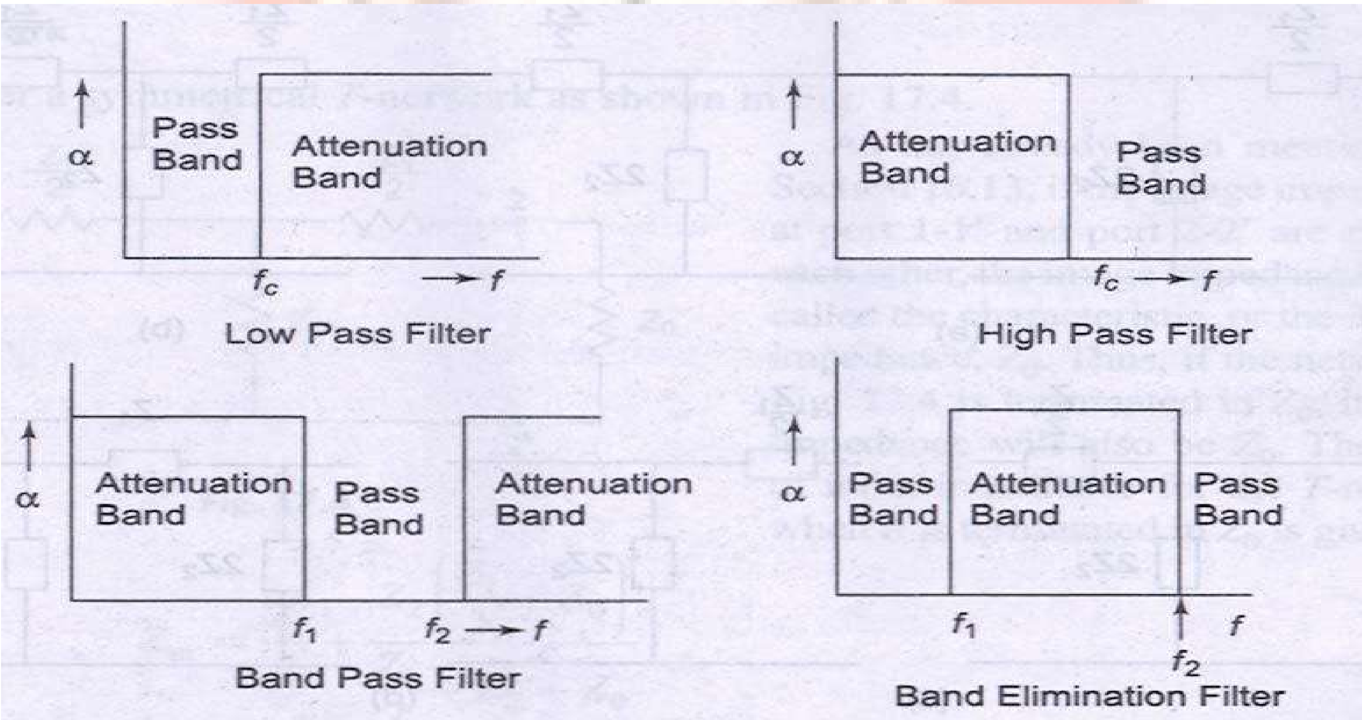


Fig.9.1(b)

HighPassFilter

A high pass (HP) filter attenuates all frequencies belowa designated cut-off frequency, f_c, and passesallfrequenciesabovef_c.Thusthepassbandof thisfilteristhefrequencyrangeabove f_c,and thestop bandisthefrequencyrangebelow f_c. Theattenuationcharacteristicof aHP filterisshown in fig.9.1 (b).

BandPassFilter

A band pass filter passes frequencies between two designated cut-off frequencies and attenuatesallotherfrequencies.ItisabbreviatedasBPfilter.Asshowninfig.9.1(b),aBPfilterhas twocut-offfrequenciesandwillhavethepassband f_2–f_1;$f_1$iscalledthelowercut–off frequency, while $f_2$is called the upper cut-off frequency.

BandEliminationfilter

Abandeliminationfilterpassesallfrequencieslyingoutsideacertain range,whileitattenuates all frequencies between the two designated frequencies. It is also referred as band stop filter. The characteristic of an ideal band elimination filter is shown in fig.9.1 (b).All frequencies between $f_1$and$f_2$will be attenuatedwhilefrequencies below $f_1$andabove$f_2$will be passed.

FILTERNETWORKS

Ideally a filter should have zero attenuation in the pass band. This condition can only be satisfied if the elements of the filter are dissipationless.which cannot be realized in practice. Filters aredesignedwithanassumptionthattheelementsof thefiltersarepurelyreactive.Filtersaremade of symmetrical T, or πsection. Tand π section can be considered as combination of unsymmetrical L sections as shown in Fig.9.2.

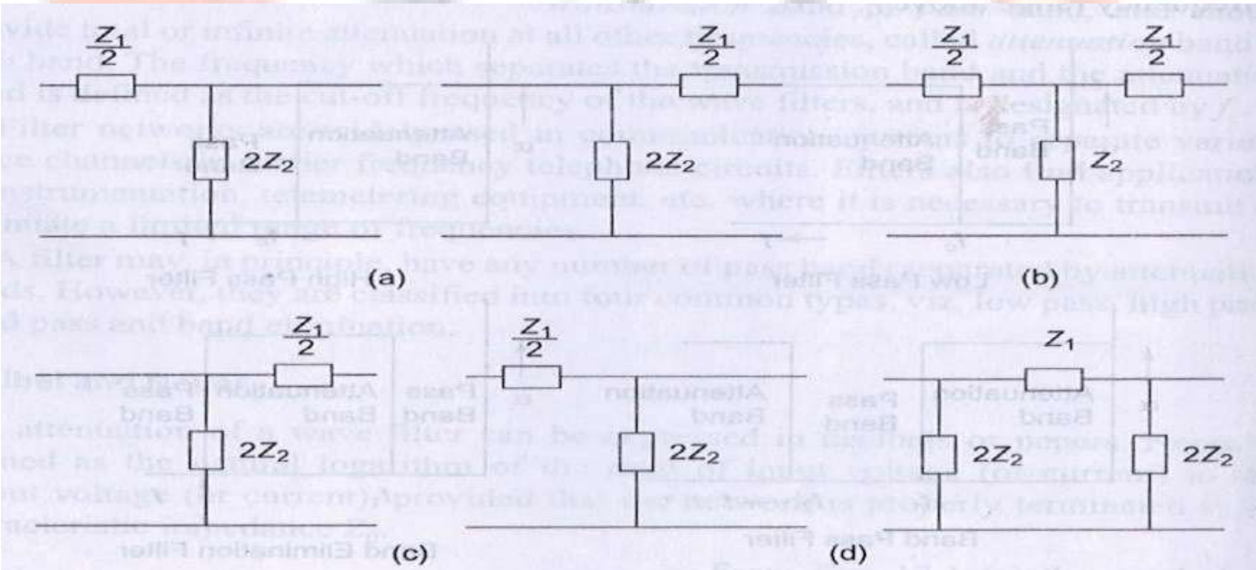


Fig. 9.2

The ladder structure is one of the commonest forms of filter network. A cascade connection ofseveralTand πsections constitutesaladdernetwork.Acommonformofthe ladder network is shown in Fig.9.3.

Figure9.3(a)representsaTsectionladdernetwork,whereasFig.9.3(b)representstheπ section laddernetwork.Itcanbeobservedthatbothnetworksareidenticalexceptattheends.

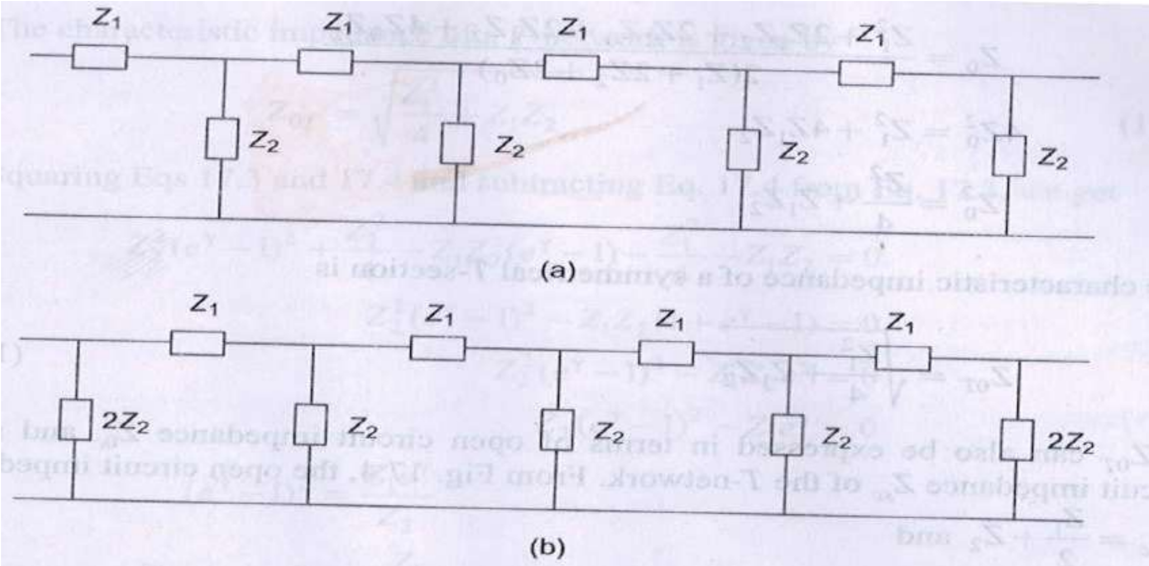


Fig. 9.3

EQUATIONSOFFILTERNETWORKS

ThestudyofthebehaviorofanyfilterrequiresthecalculationofitspropagationconstantУ, attenuationα,phaseshiftβ anditscharacteristicimpedanceZ_0.

T-Network

ConsiderasymmetricalT-networkasshowninFig. 9.4.

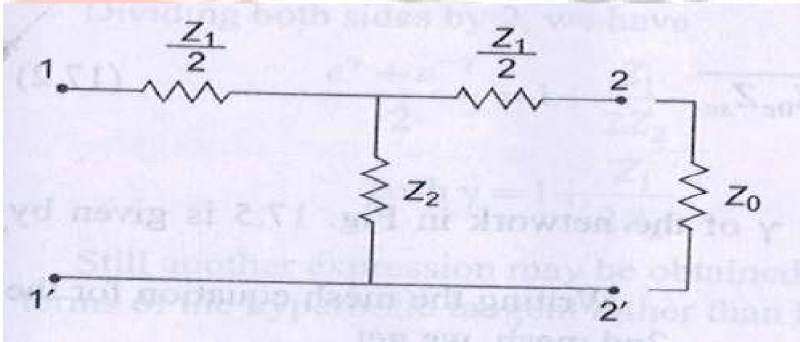


Fig.9.4

If the image impedances at port 1-1' and port 2-2' are equal to each other ,the image impedanceisthencalledthecharacteristic,ortheiterativeimpedance,Z_0.Thus,ifthenetworkin Fig.9.4is terminated inZ_0,its inputimpedance will alsobe Z_0. The value of input impedance for the T-network when it is terminated inZ_0 is given by

$$Z_{in} = \frac{Z_1}{2} + \frac{Z_2\left(\frac{Z_1}{2} + Z_0\right)}{\frac{Z_1}{2} + Z_2 + Z_0}$$

also $$Z_{in} = Z_0$$

$$\therefore \quad Z_0 = \frac{Z_1}{2} + \frac{2Z_2\left(\frac{Z_1}{2} + Z_0\right)}{Z_1 + 2Z_2 + 2Z_0}$$

$$Z_0 = \frac{Z_1}{2} + \frac{(Z_1 Z_2 + 2Z_2 Z_0)}{Z_1 + 2Z_2 + 2Z_0}$$

$$Z_0 = \frac{Z_1^2 + 2Z_1Z_2 + 2Z_1Z_0 + 2Z_1Z_2 + 4Z_0Z_2}{2(Z_1 + 2Z_2 + 2Z_0)}$$

$$4Z_0^2 = Z_1^2 + 4Z_1Z_2$$

$$Z_0^2 = \frac{Z_1^2}{4} + Z_1Z_2$$

ThecharacteristicimpedanceofasymmetricalT-sectionis

$$Z_{0T} = \sqrt{\frac{Z_1^2}{4} + Z_1Z_2}$$

(9.1)

Z_{0T}canalsobeexpressedintermsofopen circuitimpedanceZ_{0C}andshortcircuitimpedance Z_{SC} of the T – network . From Fig. 9.4, the open circuit impedance $Z_{0C} = Z_1/2 + Z_2$ and

$$Z_{sc} = \frac{Z_1}{2} + \frac{\frac{Z_1}{2} \times Z_2}{\frac{Z_1}{2} + Z_2}$$

$$Z_{sc} = \frac{Z_1^2 + 4Z_1Z_2}{2Z_1 + 4Z_2}$$

$$Z_{0c} \times Z_{sc} = Z_1Z_2 + \frac{Z_1^2}{4}$$

$$= Z_{0T}^2 \quad \text{or} \quad Z_{0T} = \sqrt{Z_{0c}Z_{sc}}$$

(9.2)

PropagationConstantofT-Network

BydefinitationthepropagationconstantУofthenetworkinFig.9.5isgivenbyУ=$\log_e I_1/I_2$

Writingthemeshequationforthe2ndmesh,we get

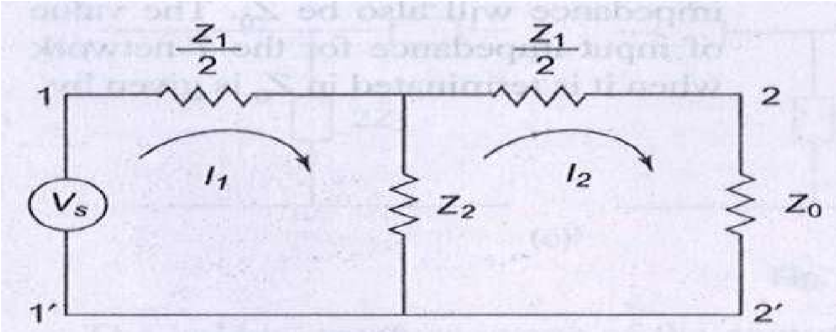


Fig.9.5

$$I_1 Z_2 = I_2\left(\frac{Z_1}{2} + Z_2 + Z_0\right)$$

$$\frac{I_1}{I_2} = \frac{\frac{Z_1}{2} + Z_2 + Z_0}{Z_2} = e^{\gamma}$$

$$\therefore \quad \frac{Z_1}{2} + Z_2 + Z_0 = Z_2 e^{\gamma}$$

$$Z_0 = Z_2(e^{\gamma} - 1) - \frac{Z_1}{2} \qquad (9.3)$$

ThecharacteristicimpedanceofaT–networkisgivenby

$$Z_{0T} = \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2} \qquad (9.4)$$

SquaringEsq.9.3and9.4andsubtractingEq.9.4fromEq.9.3,weget

$$Z_2^2(e^{\gamma}-1)^2+\frac{Z_1^2}{4}-Z_1Z_2(e^{\gamma}-1)-\frac{Z_1^2}{4}-Z_1Z_2=0$$

$$Z_2^2(e^{\gamma}-1)^2-Z_1Z_2(1+e^{\gamma}-1)=0$$

$$Z_2^2(e^{\gamma}-1)^2-Z_1Z_2e^{\gamma}=0$$

$$Z_2(e^{\gamma}-1)^2-Z_1e^{\gamma}=0$$

$$(e^{\gamma}-1)^2=\frac{Z_1e^{\gamma}}{Z_2}$$

$$e^{2\gamma}+1-2e^{\gamma}=\frac{Z_1}{Z_2e^{-\gamma}}$$

Rearrangingtheaboveequation,wehave

$$e^{-\gamma}(e^{2\gamma}+1-2e^{\gamma})=\frac{Z_1}{Z_2}$$

$$(e^{\gamma}+e^{-\gamma}-2)=\frac{Z_1}{Z_2}$$

Dividingbothsidesby2,we have

$$\frac{e^{\gamma}+e^{-\gamma}}{2}=1+\frac{Z_1}{2Z_2}$$

$$\cosh\gamma=1+\frac{Z_1}{2Z_2}$$

(9.5)

Stillanotherexpressionmayobtainedforthecomplexpropagationconstantintermsof thehyperbolic tangent rather than hyperbolic cosine.

$$\sinh \gamma = \sqrt{\cosh^2\gamma - 1}$$

$$= \sqrt{\left(1+\frac{Z_1}{2Z_2}\right)^2 - 1} = \sqrt{\frac{Z_1}{Z_1} + \left(\frac{Z_1}{2Z_2}\right)^2}$$

$$\sinh \gamma = \frac{1}{Z_2}\sqrt{Z_1 Z_2 + \frac{Z_1^2}{4}} = \frac{Z_{0T}}{Z_2}$$

(9.6)

DividingEq.9.6byEq.9.5,Weget

$$\tanh \gamma = \frac{Z_{0T}}{Z_2 + \frac{Z_1}{2}}$$

But $$Z_2 + \frac{Z_1}{2} = Z_{0c}$$

AlsofromEq. 9.2,

$$Z_{0T} = \sqrt{Z_{0c} Z_{sc}}$$

$$\tanh \gamma = \sqrt{\frac{Z_{sc}}{Z_{0c}}}$$

Also $$\sinh \frac{\gamma}{2} = \sqrt{\frac{1}{2}(\cosh \gamma - 1)}$$

Where $$\cosh \gamma = 1 + (Z_1/2Z_2)$$

$$= \sqrt{\frac{Z_1}{4Z_2}}$$ (9.7)

π– Network

Considerasymmetricalπ–sectionshowninFig.9.6.WhenthenetworkisterminatedinZ$_0$atport2 –2′itsinputimpedanceisgivenby

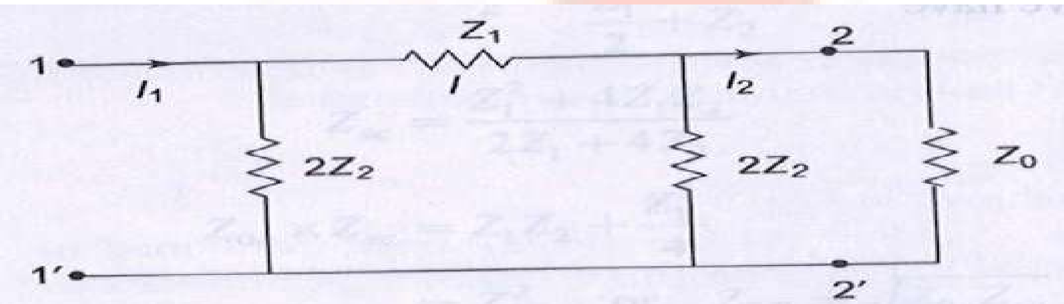


Fig.9.6

$$Z_{in} = \frac{2Z_2\left[Z_1 + \frac{2Z_2\,Z_0}{2Z_2 + Z_0}\right]}{Z_1 + \frac{2Z_2\,Z_0}{2Z_2 + Z_0} + 2Z_2}$$

By definition of characteristic impedance, $Z_{in} = Z_0$

$$Z_0 = \frac{2Z_2\left[Z_1 + \frac{2Z_2\,Z_0}{2Z_2 + Z_0}\right]}{Z_1 + \frac{2Z_2\,Z_0}{2Z_2 + Z_0} + 2Z_2}$$

$$Z_0Z_1 + \frac{2Z_2Z_0^2}{2Z_2 + Z_0} + 2Z_0Z_2 = \frac{2Z_2(2Z_1Z_2 + Z_0Z_1 + 2Z_0Z_2)}{(2Z_2 + Z_0)}$$

$$2Z_0Z_1Z_2 + Z_1Z_0^2 + 2Z_0^2Z_2 + 4Z_2^2Z_0 + 2Z_2Z_0^2 = 4Z_1Z_2^2 + 2Z_0Z_1Z_2 + 4Z_0Z_2^2$$

$$Z_1Z_0^2 + 4Z_2Z_0^2 = 4Z_1Z_2^2$$

$$Z_0^2(Z_1 + 4Z_2) = 4Z_1Z_2^2$$

$$Z_0^2 = \frac{4Z_1Z_2^2}{Z_1 + 4Z_2}$$

Rearranging the above equation leads to

$$Z_0 = \sqrt{\frac{Z_1Z_2}{1 + Z_1/4Z_2}}$$

(9.8)

which is the characteristic impedance of a symmetrical π-network,

$$Z_{0\pi} = \frac{Z_1Z_2}{\sqrt{Z_1Z_2 + Z_1^2/4}}$$

FromEq.9.1

$$Z_{0T} = \sqrt{\frac{Z_1^2}{4} + Z_1Z_2}$$

$$\therefore\ Z_{0\pi} = \frac{Z_1Z_2}{Z_{0T}}$$

(9.9)

$Z_{0\pi}$canbeexpressedintermsof theopen circuitimpedanceZ $_{0C}$andshortcircuitimpedance Z $_{SC}$of the π network shown in Fig.9.6 exclusive of the load Z $_0$.

FromFig.9.6,theinputimpedanceatport1-1′whenport2– 2′isopenisgiven by

$$Z_{0C} = \frac{2Z_2(Z_1 + 2Z_2)}{Z_1 + 4Z_2}$$

Similarly,theinputimpedanceatport1–1′whenport2–2′isshortcircuitisgivenby

$$Z_{sc} = \frac{2Z_1Z_2}{2Z_2 + Z_1}$$

$$\text{Hence} \quad Z_{0c} \times Z_{sc} = \frac{4Z_1Z_2^2}{Z_1 + 4Z_2} = \frac{Z_1Z_2}{1 + Z_1/4Z_2}$$

ThusfromEq.9.8

$$Z_{0\pi} = \sqrt{Z_{0c}\ Z_{sc}}$$

(9.10)

PropagationConstantofπ–Network

Thepropagationconstantofasymmetricalπ–section isthesameasthatforasymmetricalT–Section.

$$\text{i.e.} \qquad \cosh\gamma = 1 + \frac{Z_1}{2Z_2}$$

CLASSIFICATION OFPASSBAND AND STOP BAND

Itispossibletoverifythecharacteristicsoffiltersfrom thepropagationconstantofthe network.The propagation constant Ұ, being a function of frequency, the pass band, stop band and the cut-off point,i.e.thepointofseparationbetweenthe twobands,canbeidentified.ForsymmetricalTorπ – section, the expression for propagation constant Ұ in terms of the hyperbolic functions is given by Eqs 9.5 and 9.7 in section 9.3. From Eq.9.7, sin h Ұ/2 = √(Z $_1$/4Z_2) .

IfZ_1andZ_2arebothpureimaginaryvalues,theirratio,andhence$Z_1/4Z_2$,willbeapurereal number. Since$Z_1$and Z_2may be anywhere in the range from -j$_\alpha$to +j$_\alpha$, Z_1/ 4Z_2may also have any realvaluebetweentheinfinitelimits.Thensinhγ/2= $\sqrt{Z_1}/\sqrt{4Z_2}$willalsohaveinfinitelimits,but may be either real or imaginary depending upon whether $Z_1/4Z_2$is positive or negative.

Weknowthat thepropagationconstant isacomplexfunctionγ =α+jβ, the real partof the complexpropagationconstantα,isameasureofthe changeinmagnitudeofthecurrentor voltage in the network ,known as the attenuation constant . β is a measure of the difference in phase betweentheinputandoutputcurrentsorvoltages.KnownasphaseshiftconstantThereforeαandβ takeondifferentvaluesdependingupontheof$Z_1/4Z_2$.FromEq.9.7,Wehave

$$\sinh\frac{\gamma}{2} = \sinh\left(\frac{\alpha}{2}+\frac{j\beta}{2}\right) = \sinh\frac{\alpha}{2}\cos\frac{\beta}{2} + j\cosh\frac{\alpha}{2}\sin\frac{\beta}{2} = \sqrt{\frac{Z_1}{4Z_2}}$$

(9.11)

CaseA

IfZ_1andZ_2arethesametypeofreactances,then[$Z_1/4Z_2$]isrealandequaltosayα+x.

TheimaginarypartoftheEq.9.11mustbezero.

$$\therefore \quad \cosh\frac{\alpha}{2}\sin\frac{\beta}{2} = 0$$

(9.12)

$$\sinh\frac{\alpha}{2}\cos\frac{\beta}{2} = x$$

(9.13)

αandβmustsatisfybooththeabove equations.

Equation9.12canbesatisfiedifβ/2=0ornπ,wheren=0,1,2,.....,thencosβ/2=1andsinhα/2=x =$\sqrt{(Z_1/4Z_2)}$

Thatxshouldbealwayspositiveimplies that

$$\left|\frac{Z_1}{4Z_2}\right| > 0 \text{ and } \alpha = 2\sinh^{-1}\sqrt{\frac{Z_1}{4Z_2}}$$

(9.14)

Sinceα≠0,itindicatesthattheattenuationexists.

CaseB

ConsiderthecaseofZ$_1$andZ$_2$beingoppositetypeofreactances,i.e. $Z_1/4Z_2$isnegative, making $\sqrt{Z_1/4Z_2}$imaginary and equal to say Jx

*TherealpartoftheEq.9.11mustbezero.

$$\sinh\frac{\alpha}{2}\cos\frac{\beta}{2}=0$$

(9.15)

$$\cosh\frac{\alpha}{2}\sin\frac{\beta}{2}=x$$

(9.16)

Boththeequationsmustbesatisfiedsimultaneouslybyαandβ.Equation 9.15 maybesatisfied when $\alpha = 0$, or when $\beta = \pi$. These conditions are considered separately hereunder

(i) When$\alpha=0$;fromEq.9.15, $\sinh\alpha/2=0$.andfromEq.9.16$\sin\beta/2= x=\sqrt{(Z_1/4Z_2)}$.Butthe sinecanhave a maximum value of 1. Therefore, the above solutionis valid only for negative$Z_1/4Z_2$,andhavingmaximumvalueofunity.Itindicatestheconditionofpassbandwithzeroattenuation and follows the condition as

$$-1\leq\frac{Z_1}{4Z_2}\leq 0$$

$$\beta = 2\sin^{-1}\sqrt{\frac{Z_1}{4Z_2}} \qquad (9.17)$$

(ii) When$\beta=\pi$,fromEq.9.15,$\cos\beta/2=0$.AndfromEq.9.16,$\sin\beta/2=\pm1$;$\cosh\alpha/2=x=\sqrt{(Z_1/4Z_2)}$
.

Since$\cosh\alpha/2\geq 1$,thissolutionisvalidfornegative$Z_1/4Z_2$,andhavingmagnitude greater than, or equal to unity. It indicates the condition of stop band since $\alpha \neq 0$.

$$-\alpha\leq\frac{Z_1}{4Z_2}\leq -1$$

$$\alpha = 2\cosh^{-1}\sqrt{\frac{Z_1}{4Z_2}} \qquad (9.18)$$

It can be observed that there are three limits for case A and B. Knowing the values of $Z_1$and Z_2, it is possible to determine the case to be applied to the filter. $Z_1$and $Z_2$are made of different types of reactances, or combinations of reactances, so that, as the frequency changes, a filtermaypassfromonecasetoanother.CaseAand(ii)incaseBareattenuation bands,whereas(i) in case B is the transmission band.

Thefrequencywhichseparatestheattenuationbandfrompassbandorviceversais called cut-off frequency. The cut-off frequency is denoted by f_C, and is also termed as nominal frequency.SinceZ_0isrealinthepassbandandimaginaryinanattenuationband, $f_C$isthefrequency at which Z_0changes from being real to being imaginary. These frequencies occur at

$$\frac{Z_1}{4Z_2} = 0 \text{ or } Z_1 = 0 \qquad 9.18(a)$$

$$\frac{Z_1}{4Z_2} = -1 \text{ or } Z_1 + 4Z_2 = 0 \qquad 9.18(b)$$

Theaboveconditionscanberepresentedgraphically,asinFig.9.7.

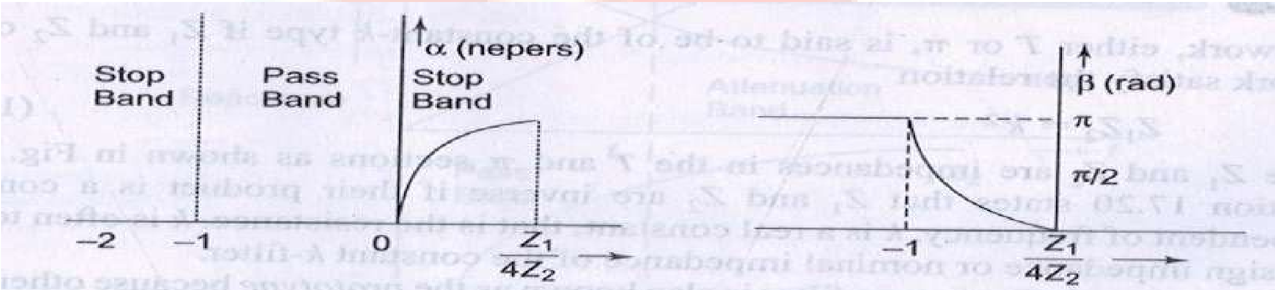


Fig. 9.7

CHARACTERISTIC IMPEDANCEIN THE PASS AND STOP BANDS

ReferringtothecharacteristicimpedanceofasymmetricalT-network,fromEq.9.1We have

$$Z_{0T} = \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2} = \sqrt{Z_1 Z_2 \left(1 + \frac{Z_1}{4Z_2}\right)}$$

IfZ_1andZ_2arepurelyreactive,let$Z_1=jx_1$and$Z_2=jx_2$,then

$$Z_{0T} = \sqrt{-x_1 x_2 \left(1 + \frac{x_1}{4x_2}\right)}$$

(9.19)

Apassbandexistswhenx$_1$andx$_2$areofoppositereactancesand

$$-1 < \frac{x_1}{4x_2} < 0$$

Substituting these conditions in Eq. 9.19, we find that $Z_{OT}$is positive and real. Now consider thestop band.Astopbandexistswhen x$_1$andx$_2$are of thesametypeofreactances; thenx$_1$/4x$_2$>0. Substituting these conditions in Eq. 9.19, we find that $Z_{OT}$is purley imaginary in this attenuation region.Another stopbandexists when x$_1$andx $_2$are of the same type of reactances, butwithx$_1$/4x$_2$ <-1.ThenfromEq.9.19,Z_{OT}isagainpurlyimaginaryintheattenuationregion.

Thus, in a pass band if a network is terminated in a pure resistance R_O(Z_{OT}= R_O), the input impedanceis$R_O$andthenetwork transmitsthepower receivedfromthesourcetothe$R_O$without any attenuation. In a stop band $Z_{OT}$is reactive. Therefore, if the network is terminated in a pure reactance (Z_O= pure reactance), the input impedance is reactive, and cannot receive or transmit power. However, the network transmits voltage and current with 90^0 phase difference and with attenuation.Ithasalreadybeenshownthatthecharacteristicsimpedanceofasymmetricalπ-sectioncanbeexpressedintermsofT.Thus,fromEq.9.9,$Z_{0\pi}$=Z_1Z_2/Z_{OT}.

SinceZ_1andZ_2arepurelyreactive,$Z_{0\pi}$isreal, ifZ_{OT}isreal andZ_{0x}isimaginaryif$Z_{OT}$is imaginary. Thus the conditions developed for T – section are valid for π – sections.

CONSTANT–KLOWPASSFILTER

Anetwork,eitherTorπ,issaidtobe oftheconstant– ktypeifZ_1andZ_2ofthenetworksatisfythe relation

$$Z_1Z_2= k^2$$

(9.20)

Where $Z_1$and $Z_2$are impedances in the T and π sections as shown in Fig.9.8.Equation 9.20 states that $Z_1$and $Z_2$are inverse if their product is a constant, independent of frequency. K is a real constantthatistheresistance.kisoften termedasdesignimpedanceornominalimpedanceofthe constant k – filter.

Theconstantk,Torπtypefilterisalsoknownasthe *prototype*becauseothermorecomplex network can be derived from it. A prototype T and π – section are shown in

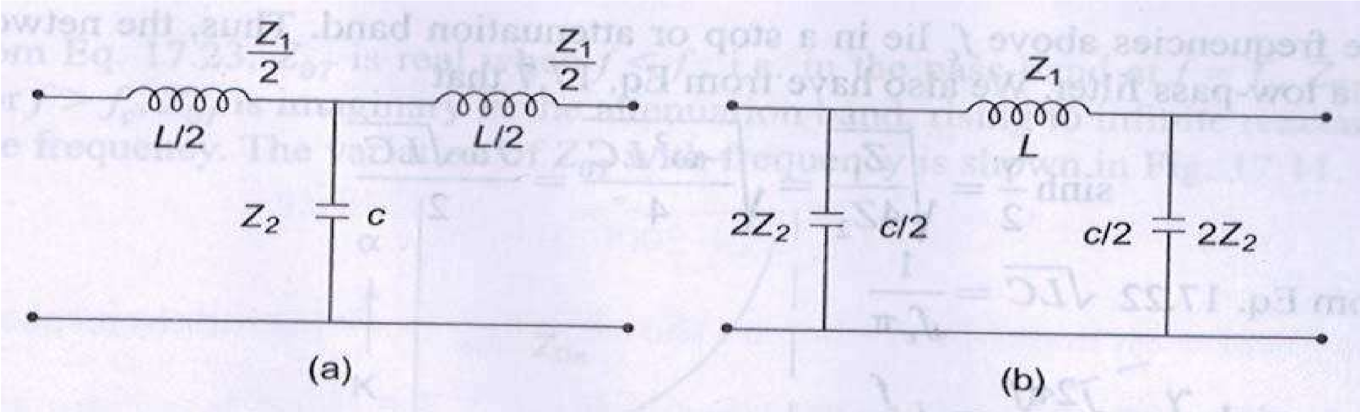


Fig.9.8

Fig.9.8(a)and(b),where Z_1=jω$_L$andZ_2=1/jω$_C$. HenceZ_1Z_2=L/C=k^2whichis independent of frequency.

$$Z_1 Z_2 = k^2 = \frac{L}{C} \quad \text{or} \quad k = \sqrt{\frac{L}{C}}$$

(9.21)

SincetheproductZ_1andZ_2isconstant,thefilterisaconstant–*k*type.FromEq.9.18(a)the cut-offfrequenciesare$Z_1/4Z_2$= 0,

$$\text{i.e.} \qquad \frac{-\omega^2 LC}{4} = 0$$

$$\text{i.e.} \qquad f = 0 \quad \text{and} \quad \frac{Z_1}{4Z_2} = -1$$

$$\frac{-\omega^2 LC}{4} = -1$$

$$\text{or} \qquad f_c = \frac{1}{\pi\sqrt{LC}}$$

(9.22)

The pass band can be determined graphically. The reactances of $Z_1$and $4Z_2$will vary with frequencyasdrawninFig.9.9.Thecut-offfrequencyat theintersectionof thecurves $Z_1$and-$4z_2$is indicated as f_C. On the X – axis as Z_1= -$4Z_2$at cut-off frequency, the pass band lies between the frequencies at which Z_1= 0, and Z_1= - $4Z_2$.

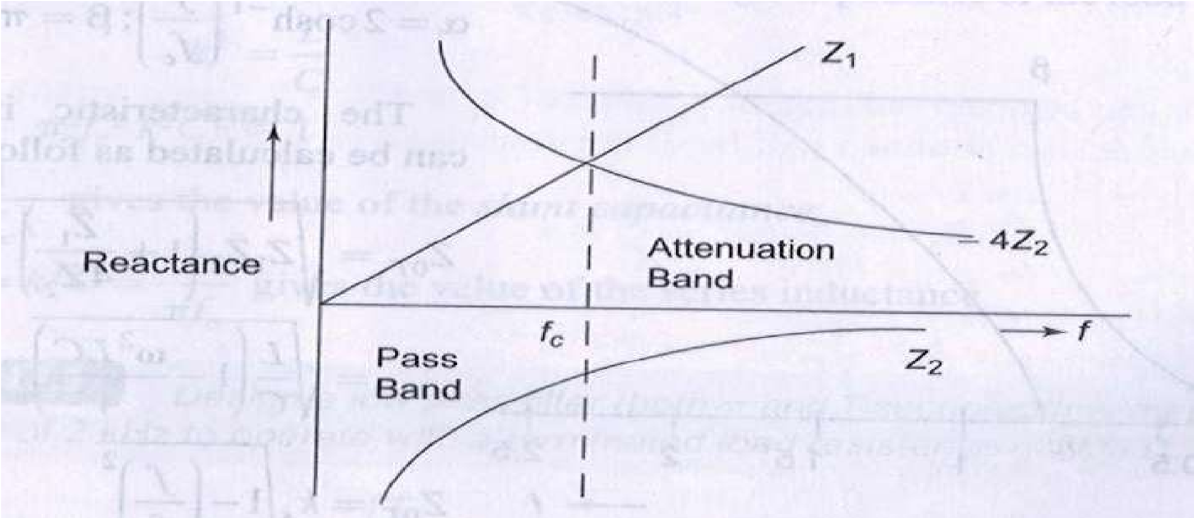


Fig.9.9

Allthefrequenciesabove$f_C$lieinastoporattenuationband,thus,the networkiscalleda low-passfilter.WealsohavefromEq.9.7that

$$\sinh\frac{\gamma}{2}=\sqrt{\frac{Z_1}{4Z_2}}=\sqrt{\frac{-\omega^2 LC}{4}}=\frac{J\omega\sqrt{LC}}{2}$$

FromEq.9.22

$$\sqrt{LC}=\frac{1}{f_c\pi}$$

$$\therefore \quad \sinh\frac{\gamma}{2}=\frac{j2\pi f}{2\pi f_c}=j\frac{f}{f_c}$$

We also know that in the pass band

$$-1<\frac{Z_1}{4Z_2}<0$$

$$-1<\frac{-\omega^2 LC}{4}<0$$

$$-1<-\left(\frac{f}{f_c}\right)^2<0$$

or $\quad \dfrac{f}{f_c}<1$

and $\quad \beta=2\sin^{-1}\left(\dfrac{f}{f_c}\right);\ \alpha=0$

In the attenuation band,

$$\frac{Z_1}{4Z_2}<-1,\ \text{i.e.}\ \frac{f}{f_c}<1 \quad f/f_c>1$$

$$\alpha=2\cosh^{-1}\left[\frac{Z_1}{4Z_2}\right]=2\cosh^{-1}\left(\frac{f}{f_c}\right);\ \beta=\pi$$

TheplotsofαandβforpassandstopbandsareshowninFig.9.10

Thus,fromFig.9.10,$\alpha=0,\beta=2\sinh^{-1}(f/f_C)forf<f_C$

$\alpha=2\cosh^{-1}(f/f_C);\beta=\pi$for$f>f_C$

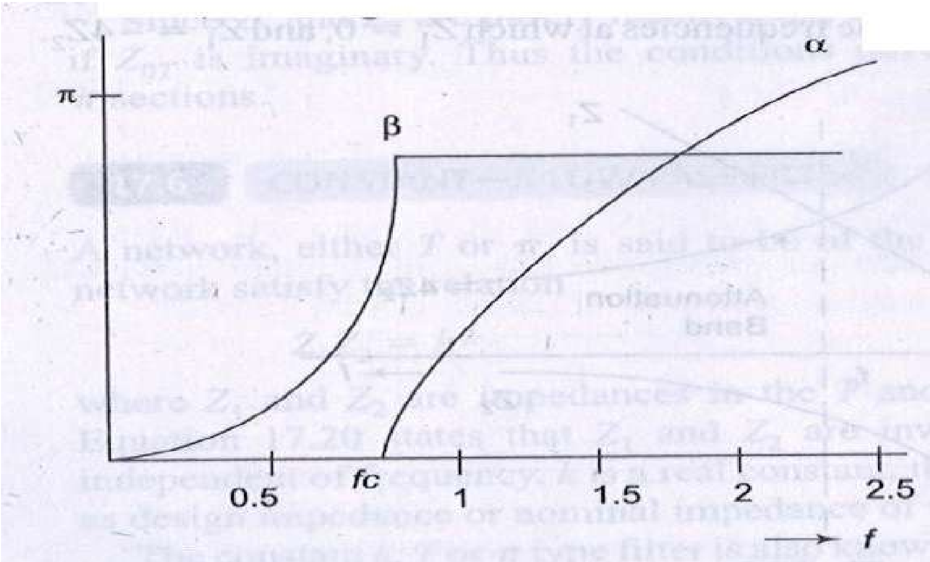


Fig.9.10

Thecharacteristicsimpedancecanbecalculatedasfollows

$$Z_{OT}=\sqrt{Z_1Z_2\left(1+\frac{Z_1}{4Z_2}\right)}$$

$$=\sqrt{\frac{L}{C}\left(1-\frac{\omega^2LC}{4}\right)}$$

$$Z_{OT}=k\sqrt{1-\left(\frac{f}{f_c}\right)^2}$$

(9.23)

From Eq.9.23, $Z_{OT}$is rael when $f<f_C$, i.e.in the pass band at $f=f_C$, Z_{OT}; and for $f>f_C$, $Z_{OT}$is imaginaryintheattenuationband,risingtoinfinitereactanceatinfinitefrequency.Thevariationof $Z_{OT}$with frequency is shown in Fig.9.11

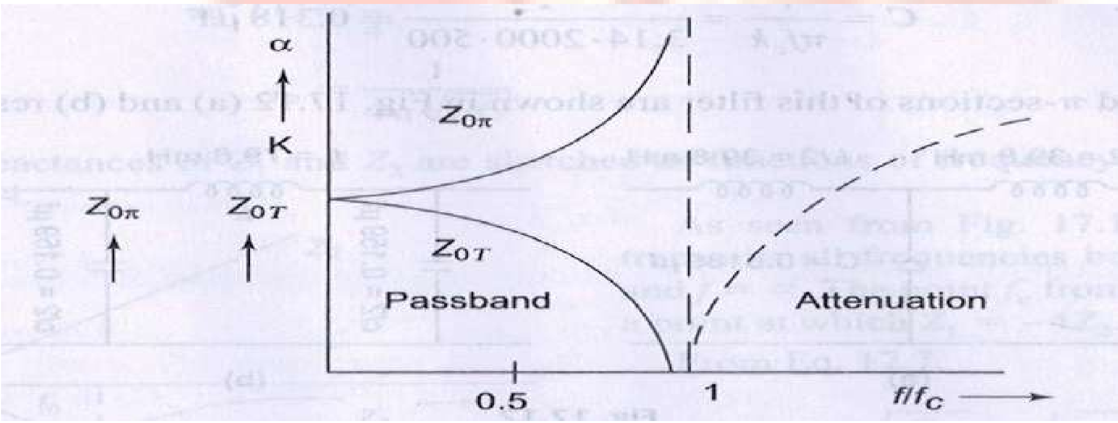


Fig.9.11

Similarly,thecharacteristicsimpedanceofaπ–networkisgivenby

$$Z_{0\pi} = \frac{Z_1 Z_2}{Z_{0T}} = \frac{k}{\sqrt{1-\left(\frac{f}{f_c}\right)^2}}$$

(9.24)

The variation of $Z_{O\pi}$with frequency is shown in Fig.9.11 . For $f < f_C$, $Z_{O\pi}$is real ; at $f = f_C$, $Z_{OT}$is infinite,andfor$f>f_C$,$Z_{O\pi}$isimaginary.Alowpassfiltercanbedesignedfromthespecificationsof cut-off frequency and load resistance.

Atcut-offfrequency,Z_1=- $4Z_2$

$$j\omega_c L = \frac{-4}{j\omega_c C}$$

$$\pi^2 f_c^2 LC = 1$$

Also we know that $k = \sqrt{L/C}$ is called the design impedance or the load resistance

$$\therefore \quad k^2 = \frac{L}{C}$$

$$\pi^2 f_c^2 k^2 C^2 = 1$$

$C = \dfrac{1}{\pi f_c k}$ gives the value of the *shunt capacitance*

and $L = k^2 C = \dfrac{k}{\pi f_c}$ gives the value of the series inductance.

Example9.1.

Designalowpassfilter(bothπandT–sections)havingacut-offfrequencyof2*kHz* to operate with a terminated load resistance of 500 Ω

.**solution**.Itisgiventhat$k=\sqrt{(L/C)}$=500 Ω,andf_C=2000Hz we

know that L = $k/\pi f_C$= 500/3.14 x 2000 = 79.6 mH

C=1/$\pi f_C k$=1/3.14.2000.500=0.318μF

TheTandπ–sectionsofthisfilterareshowninFig.9.12(a)and(b)respectively.

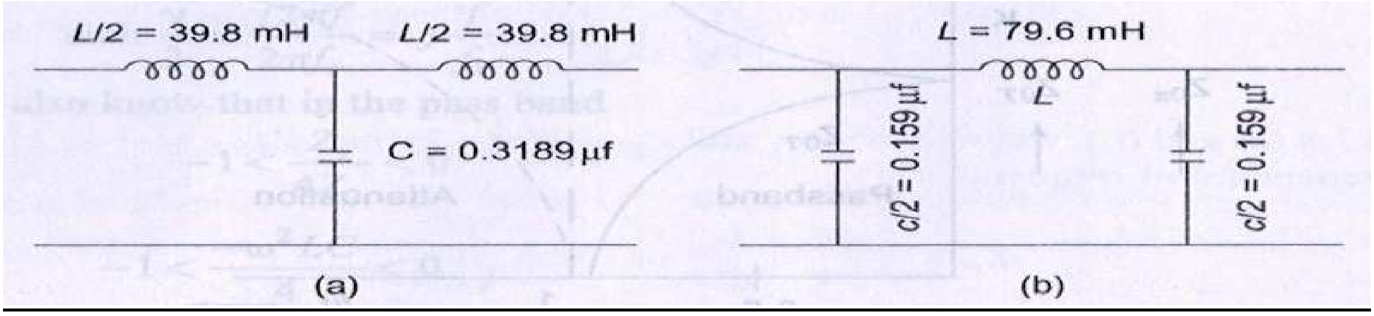


Fig.9.12

CONSTANTK–HIGHPASSFILTER

Constant K – high pass filter can be obtained by changing the positions of series and shunt arms of thenetworksshowninFig.9.8.TheprototypehighpassfiltersareshowninFig.9.13,whereZ_1=-j/$\omega_C$andZ_2= jωL .

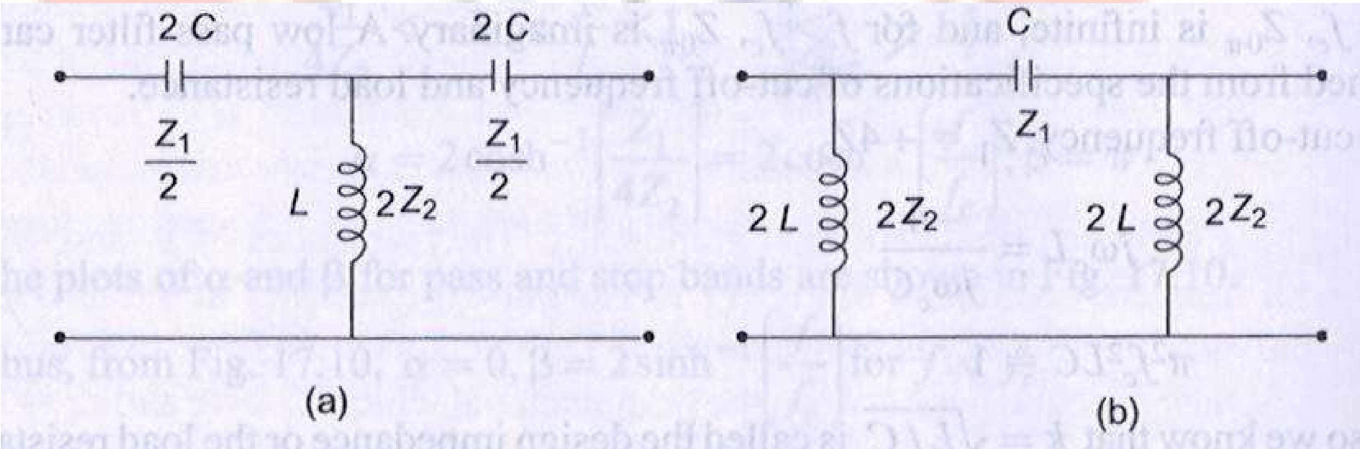


Fig.9.13

Again,itcanbeobservedthattheproductofZ_1andZ_2isindependentoffrequency,andthe filter design obtained will be of the constantk type .Thus, $Z_1Z_2$are given by

$$Z_1Z_2 = \frac{-j}{\omega C}\, j\omega L = \frac{L}{C} = k^2$$

$$k = \sqrt{\frac{L}{C}}$$

Thecut-offfrequenciesaregivenbyZ_1=0andZ_2=-4Z_2.

Z_1=0indicatesj/ωC=0,orω→α

FromZ_1=-4Z_2

$$-j/\omega C = -4j\omega L$$

$$\omega^2 LC = 1/4$$

$$\text{or} \qquad f_c = \frac{1}{4\pi\sqrt{LC}}$$

(9.25)

ThereactancesofZ_1andZ_2aresketchedasfunctionsoffrequencyasshowninFig.9.14.

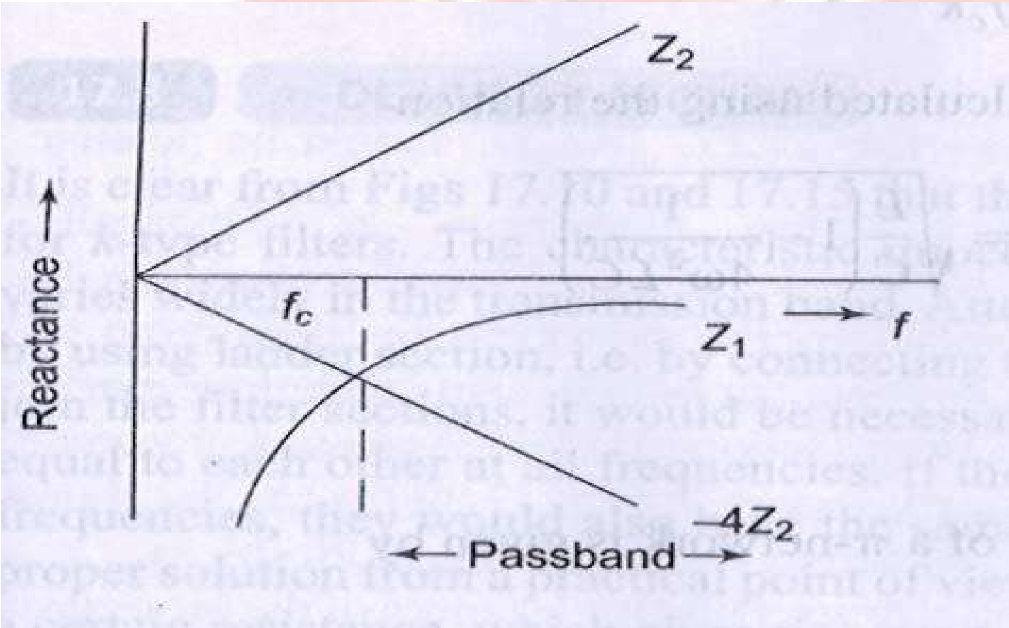


Fig.9.14

AsseenfromFig.9.14,thefiltertransmitsallfrequenciesbetween$f$=$f_C$and$f$=α.Thepointf_C fromthegraphisapointatwhichZ_1=-4Z_2.From

Eq.9.7,

$$\sinh\frac{\gamma}{2} = \sqrt{\frac{Z_1}{4Z_2}} = \sqrt{\frac{-1}{4\omega^2 LC}}$$

FromEq.9.25,

$$f_c = \frac{1}{4\pi\sqrt{LC}}$$

$$\therefore \quad \sqrt{LC} = \frac{1}{4\pi f_c}$$

$$\therefore \quad \sinh\frac{\gamma}{2} = \sqrt{\frac{-(4\pi)^2(f_c)^2}{4\omega^2}} = j\frac{f_c}{f}$$

Inthepassband, $-1<Z_1/4Z_2<0$,$\alpha=0$ortheregioninwhich$f_C/f<1$isapassband$\beta=2\sin^{-1}(f_C/f)$

Intheattenuationband$Z_1/4Z_2<-1$,i.e.$f_C/f>1$

$\alpha=2\cosh^{-1}[Z_1/4Z_2]$

$=2\cos^{-1}(f_C/f)$;$\beta=-\pi$

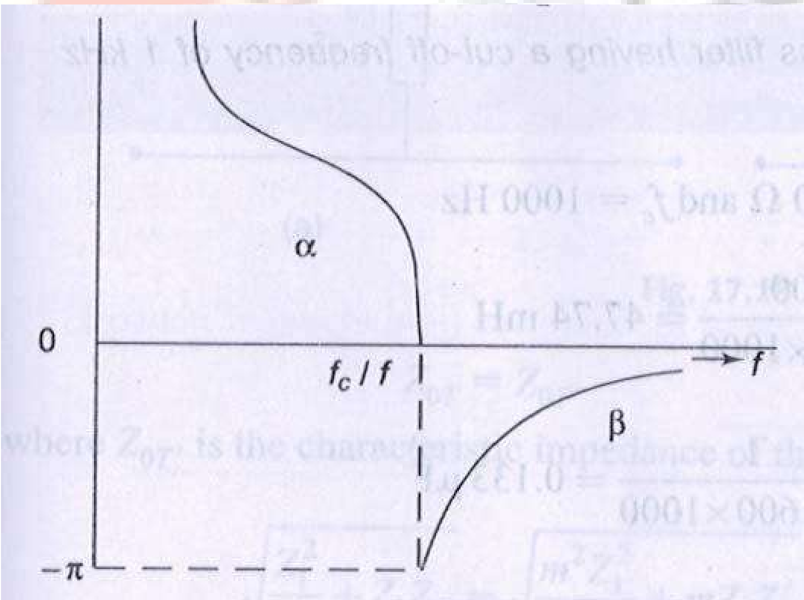


Fig.9.15

TheplotsofαandβforpassandstopbandsofahighpassfilternetworkareshowninFig.9.15.

Ahighpassfiltermaybe designedsimilartothelowpassfilterbychoosingaresistiveload requalto the constant k , such that $R = k = \sqrt{L/C}$

$$f_c = \frac{1}{4\pi\sqrt{L/C}}$$

$$f_c = \frac{k}{4\pi L} = \frac{1}{4\pi Ck}$$

Since $\sqrt{C} = \frac{L}{k}$,

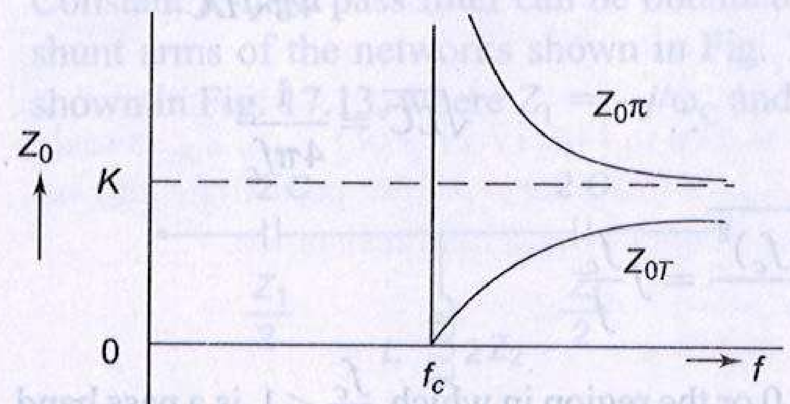
$$L = \frac{k}{4\pi f_c} \text{ and } C = \frac{1}{4\pi f_c k}$$

Thecharacteristicimpedancecanbecalculatedusingtherelation

$$Z_{0T} = \sqrt{Z_1 Z_2\left(1 + \frac{Z_1}{4Z_2}\right)} = \sqrt{\frac{L}{C}\left(1 - \frac{1}{4\omega^2 LC}\right)}$$

$$Z_{0T} = k\sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

Similarly,thecharacteristicimpedanceofaπ–networkisgivenby



$$Z_{0\pi} = \frac{Z_1 Z_2}{Z_{0T}} = \frac{k^2}{Z_{0T}} = \frac{k}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$

(9.26)

Fig.9.16

Theplotofcharacteristicimpedanceswithrespecttofrequencyisshownin Fig.9.16.

Example9.2.

Designahighpassfilterhavingacut-offfrequencyof1kHzwithaloadresistance of600Ω.

Solution. ItisgiventhatR$_L$=K=600Ωandf_C=1000Hz L = K /4πf_c= 600 /4 x π x 1000 = 47.74 mH

C=1/4πkf_C=1/4πx600x1000=0.133µF

TheTandπ–sectionsofthefilterareshowninFig.9.17.

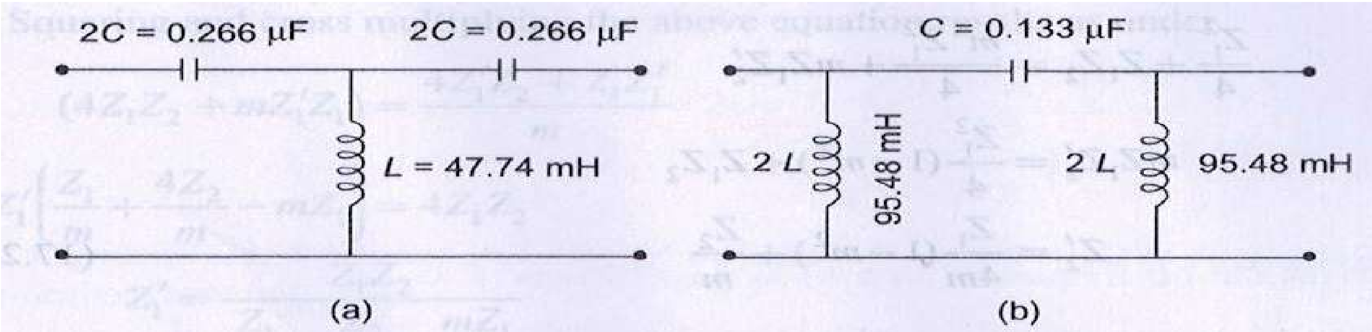


Fig.9.17

m–DERIVEDT–SECTIONFILTER

ItisclearfromFigs.9.10and9.15thattheattenuation isnotsharpinthe stop bandfor k-typefilters. The characteristic impedance, $Z_0$is a function of frequency and varies widely in the transmission band. Attenuation can be increased in the stop band by using ladder section, i.e.by connecting two or more identical sections. In order to join the filter sections, it would be necessary that their characteristic impedances be equal to each other at all frequencies. If their characteristic impedances match at all frequencies, they would also have the same pass band .However , cascading is not a proper solution from a practical point of view .

This is because practical elements have a certain resistance, which gives rise to attenuation in the pass band also. Therefore, any attempt to increase attenuation in stop band by cascading also results in an increase of 'α' in the pass band .If the constant k section is regarded as the prototype, it is possible to design a filter to have rapid attenuation in the stop band , and the samecharacteristicimpedanceastheprototypeatallfrequencies.Suchafilteriscalled*m–derived filter*.SupposeaprototypeT –networkshown inFig.9.18(a)hastheseriesarmmodifiedasshownin Fig.9.18 (b) , where *m* is a constant . Equating the characteristic impedance of the networks in Fig.9.18, we have

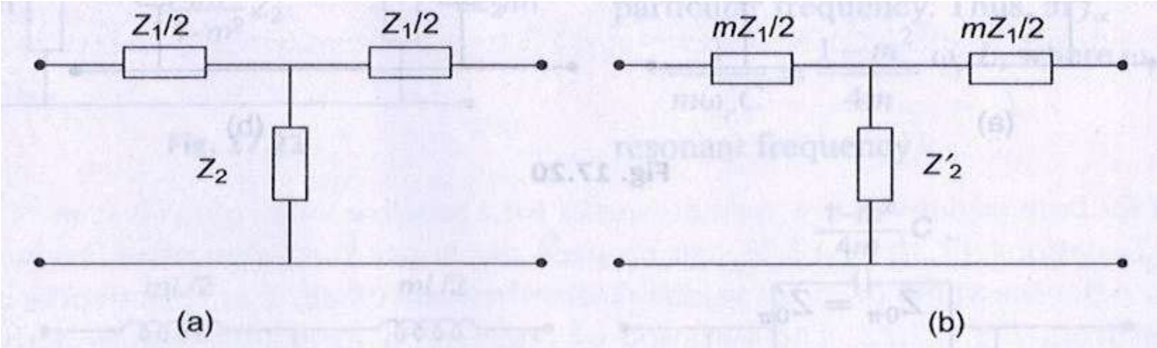


Fig.9.18

$Z_{OT}=Z_{OT}'$

Where Z_{OT}, is the characteristic impedance of the modified (m–derived) T–network.

$$\sqrt{\frac{Z_1^2}{4}+Z_1Z_2}=\sqrt{\frac{m^2Z_1^2}{4}+mZ_1Z_2'}$$

$$\frac{Z_1^2}{4}+Z_1Z_2=\frac{m^2Z_1^2}{4}+mZ_1Z_2'$$

$$mZ_1Z_2'=\frac{Z_1^2}{4}(1-m^2)+Z_1Z_2$$

$$Z_2'=\frac{Z_1}{4m}(1-m^2)+\frac{Z_2}{m}$$

(9.27)

It appears that the shunt arm Z'_2 consists of two impedances in series as shown in Fig.9.19.

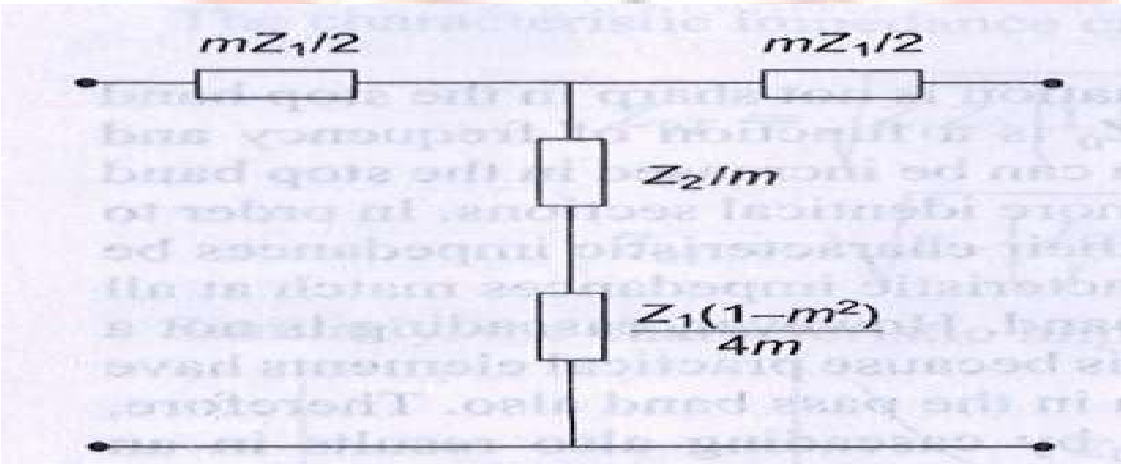


Fig.9.19

From Eq. 9.27, $1-m^2/4m$ should be positive to realize the impedance Z'_2 physically, i.e. $0<m<1$. Thus m–derived section can be obtained from the prototype by modifying its series and shunt arms. The same technique can be applied to π section network. Suppose a prototype π – network shown in Fig. 9.20 (a) has the shunt arm modified as shown in Fig. 9.20(b).

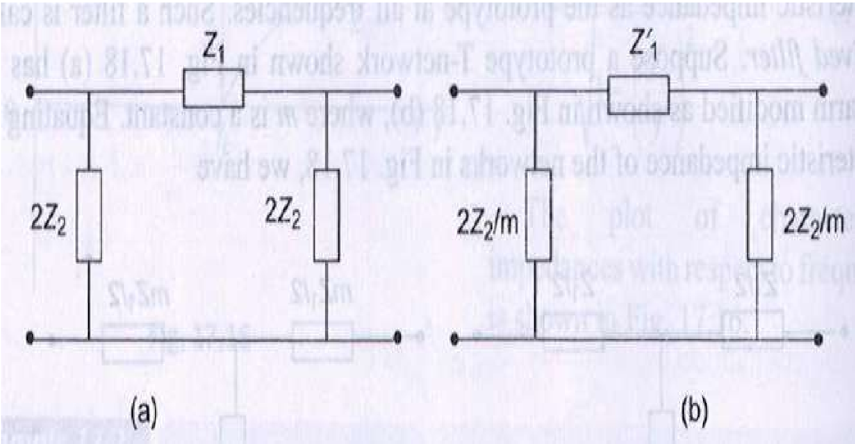


Fig.9.20

$$Z_{0\pi}=Z'_{0\pi}$$

Where $Z'_{0\pi}$ is the characteristic impedance of the modified (m–derived) π–network.

$$\therefore \quad \sqrt{\frac{Z_1Z_2}{1+\frac{Z_1}{4Z_2}}}=\sqrt{\frac{Z_1'\frac{Z_2}{m}}{1+\frac{Z_1'}{4\cdot Z_2/m}}}$$

Squaringandcrossmultiplyingtheaboveequationresultsasunder.

$$(4Z_1Z_2 + mZ_1'Z_1) = \frac{4Z_1'Z_2 + Z_1Z_1'}{m}$$

$$Z_1'\left(\frac{Z_1}{m} + \frac{4Z_2}{m} - mZ_1\right) = 4Z_1Z_2$$

or $$Z_1' = \frac{Z_1Z_2}{\frac{Z_1}{4m} + \frac{Z_2}{m} - \frac{mZ_1}{4}}$$

$$= \frac{Z_1Z_2}{\frac{Z_2}{m} + \frac{Z_1}{4m}(1-m^2)}$$

$$Z_1' = \frac{Z_1Z_2\frac{4m^2}{(1-m^2)}}{\frac{Z_2 4m^2}{m(1-m^2)} + Z_1m} = \frac{mZ_1\frac{Z_2 4m}{(1-m^2)}}{mZ_1 + \frac{Z_2 4m}{(1-m^2)}}$$

(9.28)

Itappearsthattheseriesarmofthem – derivedπsectionisaparallelcombinationof $mZ_1$and $4mZ_2/1 - m^2$. The derived m section is shown in Fig.9.21.

m–Derived LowPassFilter

InFig.9.22,both*m*–derivedlowpassTand πfiltersectionsareshown.For the T –section shownin Fig.9.22(a) , the shunt arm is to be chosen so that it is resonant at some frequency $f_\alpha$above cut-off frequency f_C.

If the shuntarm is series resonant ,its impedance willbe minimum or zero .Therefore , the outputiszeroandwillcorrespondtoinfiniteattenuationatthisparticularfrequency.Thus,atf_α

$1/m\omega_rC=1–m^2/4m\omega_rL$,where$\omega_r$istheresonantfrequency

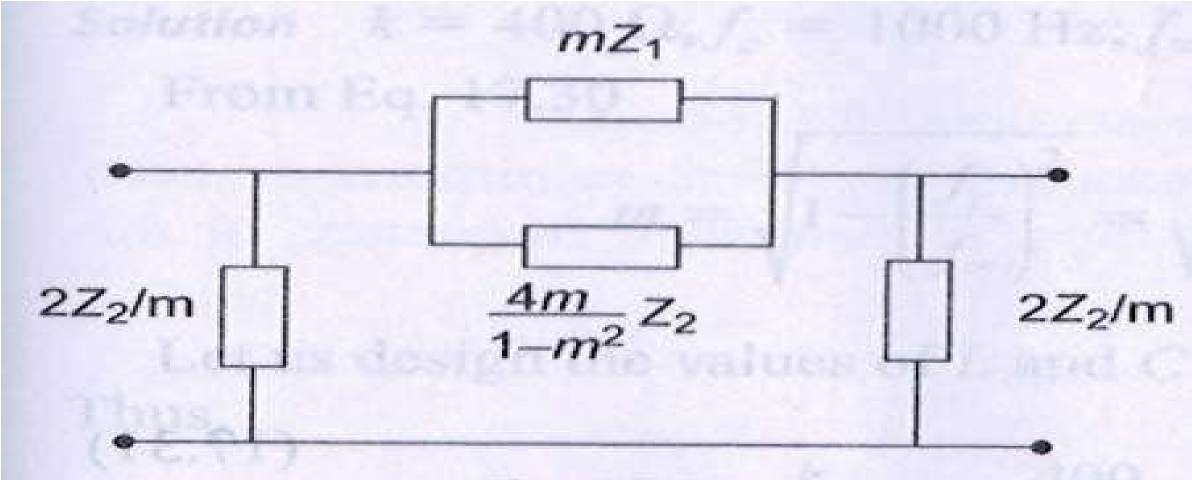


Fig.9.21

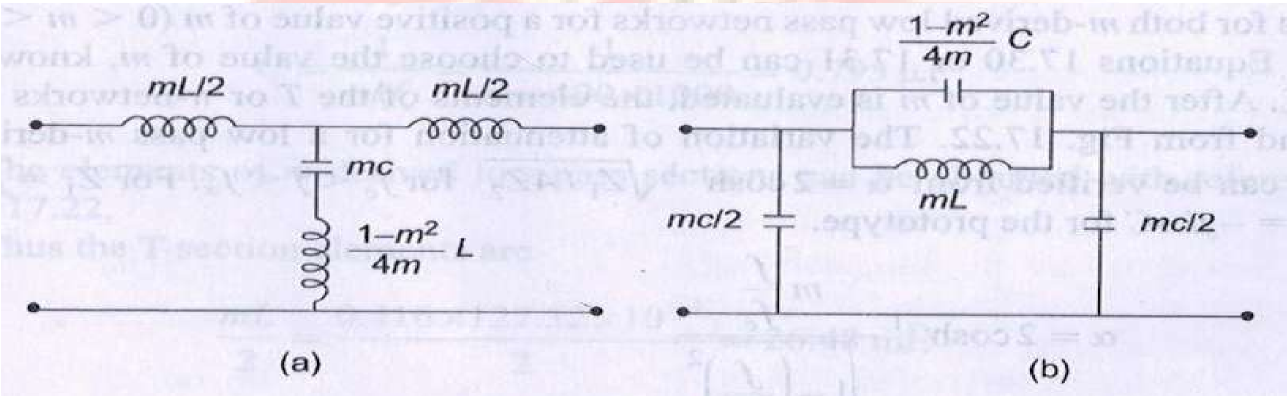


Fig.9.22

$$\omega_r^2 = \frac{4}{(1-m^2)LC}$$

$$f_r = \frac{1}{\pi\sqrt{LC(1-m^2)}} = f_\infty$$

Sincethecut-offfrequencyforthelowpassfilteris $f_c = 1/\pi\sqrt{LC}$

$$f_\infty = \frac{f_c}{\sqrt{1-m^2}}$$

(9.29)

$$\text{or} \qquad m = \sqrt{1-\left(\frac{f_c}{f_\infty}\right)^2}$$

(9.30)

If a sharp cut-off is desired, f_α should be near to f_c. From Eq.9.29, it is clear that for the smaller the value of m, f_α comes close to f_c. Equation 9.30 shows that if f_c and f_α are specified , the necessary value of m may then be calculated. Similarly, for m – derived π section, the inductance andcapacitance intheseriesarmconstitutearesonantcircuit.Thus,at f_α a frequencycorresponds to infinite attenuation, i.e. at f_α

$$m\omega_r L = \frac{1}{\left(\frac{1-m^2}{4m}\right)\omega_r C}$$

$$\omega_r^2 = \frac{4}{LC(1-m^2)}$$

$$f_r = \frac{1}{\pi\sqrt{LC(1-m^2)}}$$

$$\text{Since,} \quad f_c = \frac{1}{\pi\sqrt{LC}}$$

$$f_r = \frac{f_c}{\sqrt{1-m^2}} = f_\infty$$

(9.31)

Thusforboth m–derivedlowpassnetworksforapositivevalueof$m$($0<m<1$),$f_\alpha>f_c$. Equations 9.30 or 9.31 can beusedto choose thevalueof m,knowing$f_c$andf_r.After thevalue of m is evaluated, the elements of the T or π – networks can be found from Fig.9.22. The variation of attenuationfor alowpass$m$– derivedsectioncanbe verifiedfrom$\alpha=2\cosh^{-1}\sqrt{Z_1/4Z_2}forf_c<f<f_\alpha$. For $Z_1=j\omega L$ and $Z_2=-j/\omega C$for the prototype.

$$\therefore \qquad \alpha = 2\cosh^{-1}\frac{m\dfrac{f}{f_c}}{\sqrt{1-\left(\dfrac{f}{f_\infty}\right)^2}}$$

$$\text{and} \qquad \beta = 2\sin^{-1}\sqrt{\left|\frac{Z_1}{4Z_1}\right|} = 2\sin^{-1}\frac{m\dfrac{f}{f_c}}{\sqrt{1-\left(\dfrac{f}{f_c}\right)^2(1-m)^2}}$$

Figure9.23showsthevariationofα,βandZ_0withrespectto frequencyfor an$m$–derived low pass filter.

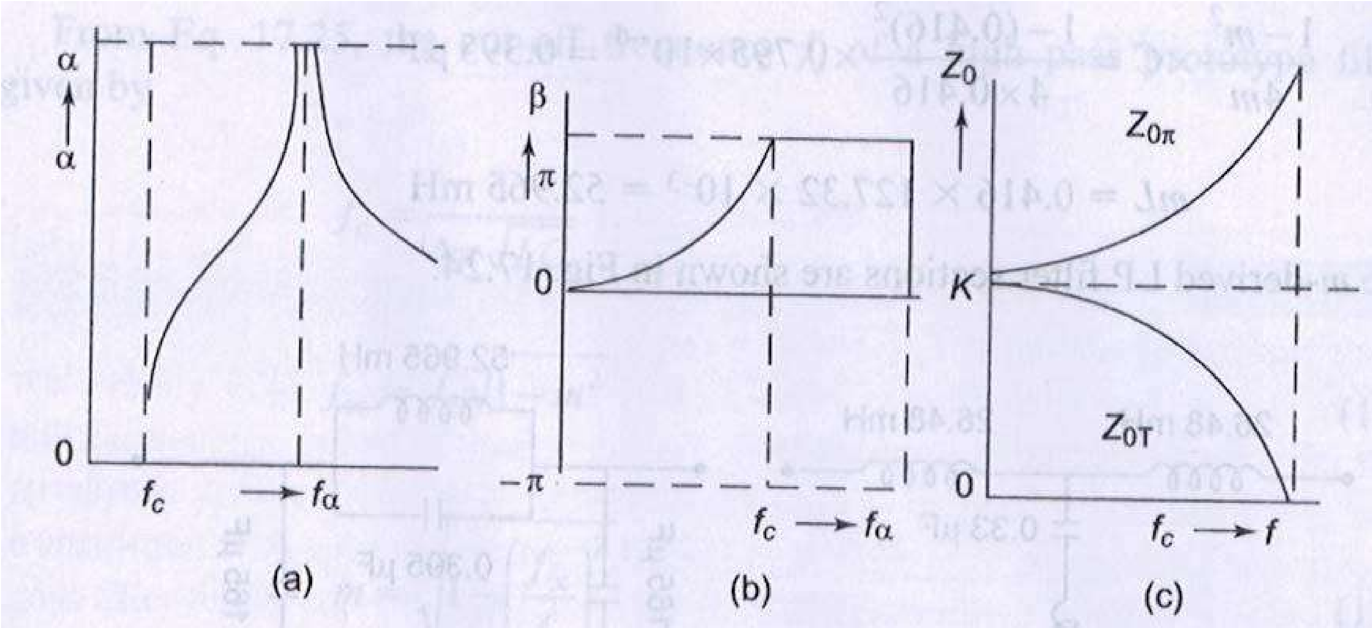


Fig.9.23

Example9.3

Designam–derivedlowpassfilterhavingcut-offfrequencyof1kHz, design impedanceof400Ω,andtheresonantfrequency1100Hz.

Solution.k=400Ω,f_C=1000Hz;f_α=1100Hz From

Eq.9.30

$$m=\sqrt{1-\left(\frac{f_c}{f_\infty}\right)^2}=\sqrt{1-\left(\frac{1000}{1100}\right)^2}=0.416$$

LetusdesignthevaluesofL andCfora lowpass,K –typefilter(prototypefilter). Thus,

$$L=\frac{k}{\pi f_c}=\frac{400}{\pi\times1000}=127.32 \text{ mH}$$

$$C=\frac{1}{\pi k f_c}=\frac{1}{\pi\times400\times1000}=0.795\ \mu\text{F}$$

Theelementsof*m*–derivedlowpasssectionscanbe obtainedwithreferencetoFig.9.22.

ThustheT-sectionelementsare

$$\frac{mL}{2} = \frac{0.416\times127.32\times10^{-3}}{2} = 26.48\ \text{mH}$$

$$mC = 0.416 \times 0.795 \times 10^{-6} = 0.33\ \mu\text{F}$$

$$\frac{1-m^2}{4m}L = \frac{1-(0.416)^2}{4\cdot0.416}\times127.32\times10^{-3} = 63.27\ \text{mH}$$

The π-section elements are

$$\frac{mC}{2} = \frac{0.416\times0.795\times10^{-6}}{2} = 0.165\ \mu\text{F}$$

$$\frac{1-m^2}{4m}\times C = \frac{1-(0.416)^2}{4\times0.416}\times0.795\times10^{-6} = 0.395\ \mu\text{F}$$

$$mL = 0.416 \times 127.32 \times 10^{-3} = 52.965\ \text{mH}$$

The*m*–derivedLPfiltersectionsareshowninFig.9.24.

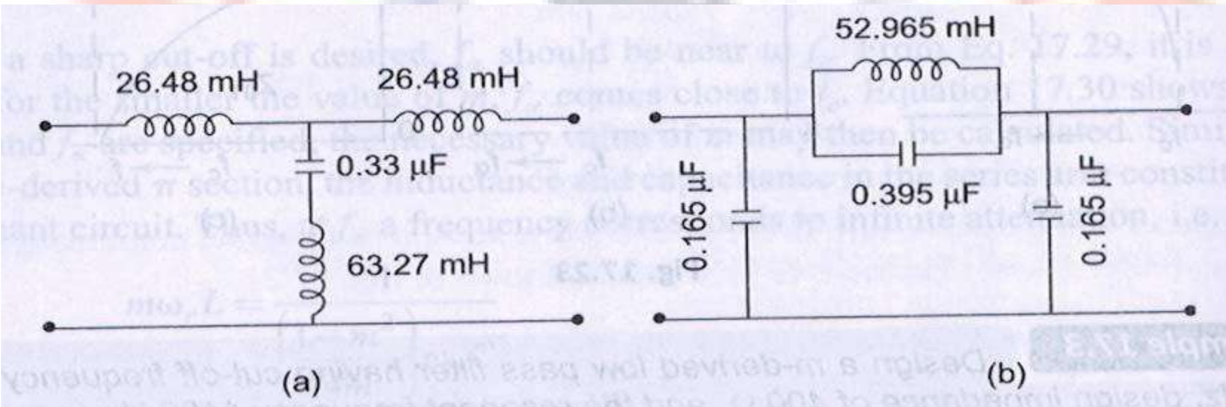


Fig.9.24

***m*–Derived High Pass Filter**

InFig.9.25both*m*–derivedhighpassTandπ–sectionare shown.

If the shunt arm in T – section is series resonant, it offers minimum or zero impedance.Therefore,theoutputiszeroand,thus,atresonancefrequencyorthefrequency corresponds to infinite attenuation.

$$\omega_r \frac{L}{m} = \frac{1}{\omega_r \dfrac{4m}{1-m^2} C}$$

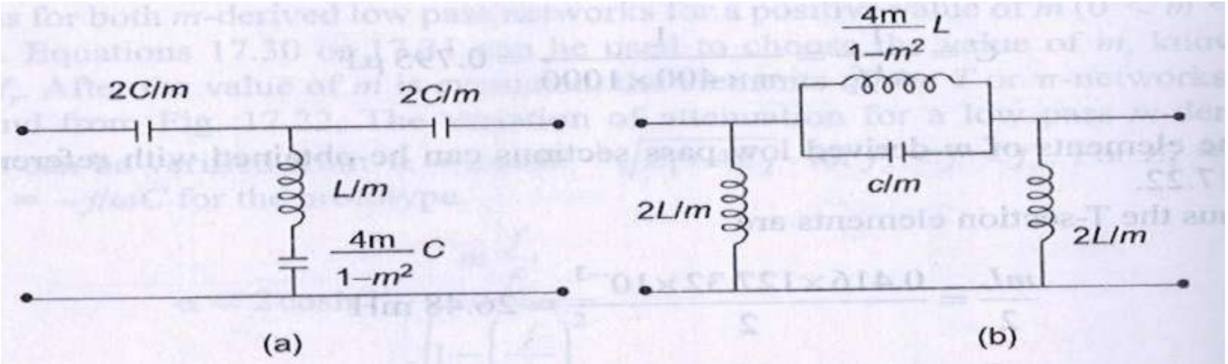


Fig.9.25

$$\omega_r^2 = \omega_\infty^2 = \frac{1}{\dfrac{L}{m}\dfrac{4m}{1-m^2}C} = \frac{1-m^2}{4LC}$$

$$\omega_\infty = \frac{\sqrt{1-m^2}}{2\sqrt{LC}} \text{ or } f_\infty = \frac{\sqrt{1-m^2}}{4\pi\sqrt{LC}}$$

FromEq.9.25,thecut–offfrequency$f_C$ofahighpassprototypefilterisgivenby

$$f_c = \frac{1}{4\pi\sqrt{LC}}$$

$$f_\infty = f_c\sqrt{1-m^2}$$

(9.32)

$$m = \sqrt{1-\left(\frac{f_\infty}{f_c}\right)^2}$$

(9.33)

Similarly,forthe*m*–derivedπ–section,theresonantcircuitisconstitutedbythe series arm inductance and capacitance . Thus , at f_α

$$\frac{4m}{1-m^2}\omega_r L = \frac{1}{\frac{\omega_r}{m} C}$$

$$\omega_r^2 = \omega_\infty^2 = \frac{1-m^2}{4LC}$$

$$\omega_\infty = \frac{\sqrt{1-m^2}}{2\sqrt{LC}} \quad \text{or} \quad f_\infty = \frac{\sqrt{1-m^2}}{4\pi\sqrt{LC}}$$

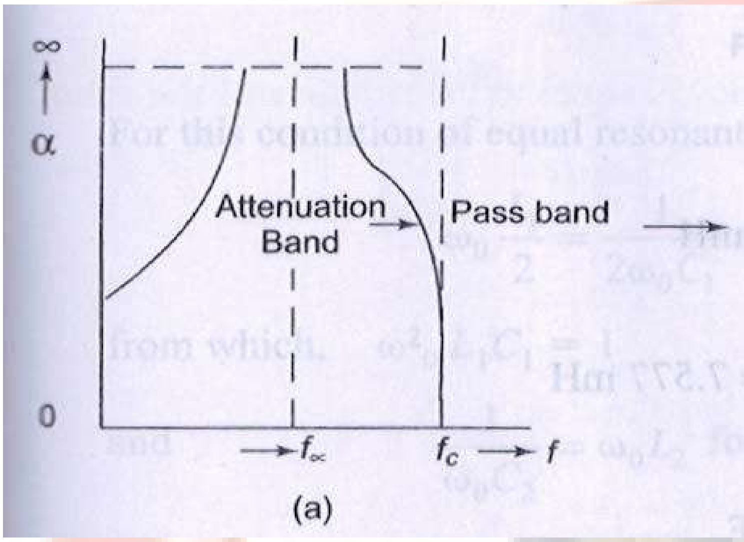


Fig.9.26

Thusthefrequencycorrespondingtoinfiniteattenuationisthesameforbothsections. Equation 9.33may be used todetermine m fora given $f_\alpha$and f_C. The elements of the m–derivedhighpass$T$or π–sectionscan befoundfrom Fig.9.25.Thevariationofα,βand$Z_0$with frequency is shown in Fig. 9.26.

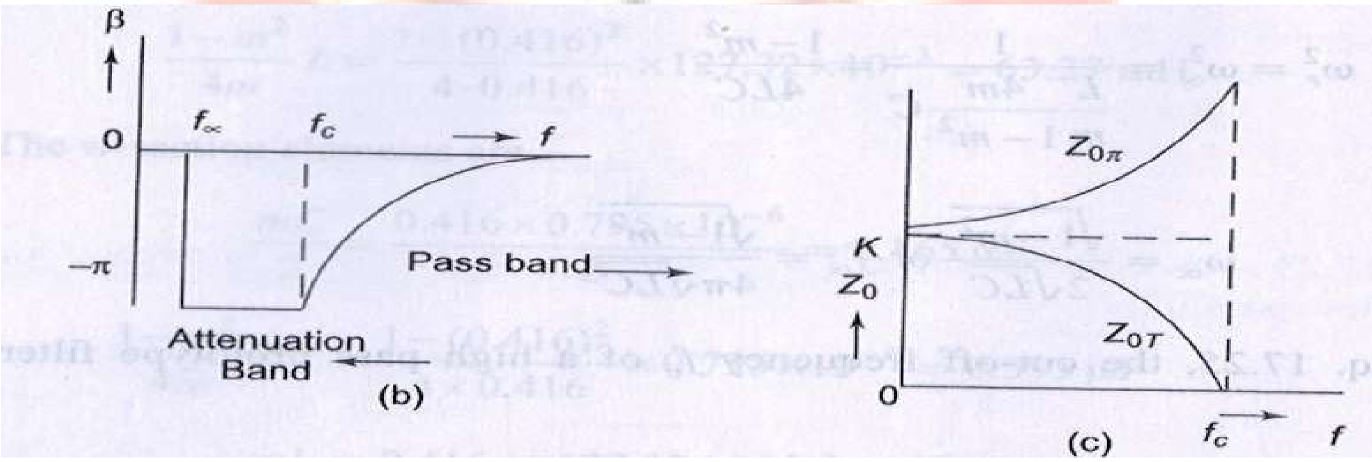


Fig.9.26

Example 9.4.

Designam-derivedhighpassfilterwithacut-offfrequencyof10kHz; design impedanceof5Ωandm= 0.4.

Solution. Fortheprototypehighpassfilter,

$$L = \frac{k}{4\pi f_c} = \frac{500}{4\times\pi\times 10000} = 3.978\ \text{mH}$$

$$C = \frac{1}{4\pi k f_c} = \frac{1}{4\pi\times 500\times 10000} = 0.0159\ \mu\text{F}$$

Theelementsofm-derivedhighpasssectionscanbe obtainedwithreferencetoFig.9.25.Thus, theT-sectionelementsare

$$\frac{2C}{m} = \frac{2\times 0.0159\times 10^{-6}}{0.4} = 0.0795\ \mu\text{F}$$

$$\frac{L}{m} = \frac{3.978\times 10^{-3}}{0.4} = 9.945\ \text{mH}$$

$$\frac{4m}{1-m^2}C = \frac{4\times 0.4}{1-(0.4)^2}\times 0.0159\times 10^{-6} = 0.0302\ \mu\text{F}$$

The π-section elements are

$$\frac{2L}{m} = \frac{2\times 0.0159\times 10^{-3}}{0.4} = 19.89\ \text{mH}$$

$$\frac{4m}{1-m^2}\times L = \frac{4\times 0.4}{1-(0.4)^2}\times 3.978\times 10^{-3} = 7.577\ \text{mH}$$

$$\frac{C}{m} = \frac{0.0159}{0.4}\times 10^{-6} = 0.0397\ \mu\text{F}$$

Tand πsectionsofthem–derivedhighpassfilterareshowninFig.9.27.

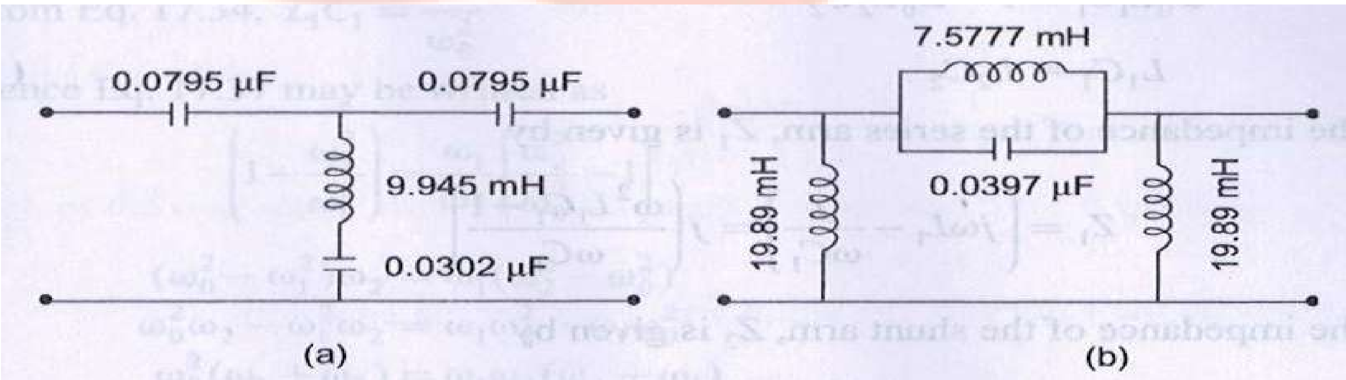


Fig.9.27

BANDPASSFILTER

AsalreadyexplainedinSection 9.1,abandpassfilterisonewhichattenuatesallfrequenciesbelow a lower cut-off frequency $f_1$and above an upper cut-off frequencyf_2.Frequencieslyingbetween $f_1$and $f_2$comprise the pass band ,and are transmitted with zero attenuation .A band pass filter may beobtainedbyusingalowpassfilterfollowedbyahighpassfilterinwhichthecut-offfrequency of theLP filterisabovethecut-offfrequency ofthe HP filter , theoverlapthusallowing onlyabandof frequencies to pass . This is not economical in practice; it is more economical to combine the low and high pass functions into a single filter section .

Consider the circuit in Fig.9.28, each arm has a resonant circuit with same resonant frequency,i.e.theresonantfrequencyof theseriesarmandtheresonantfrequencyof theshunt arm are made equal to obtain the band pass characteristic.

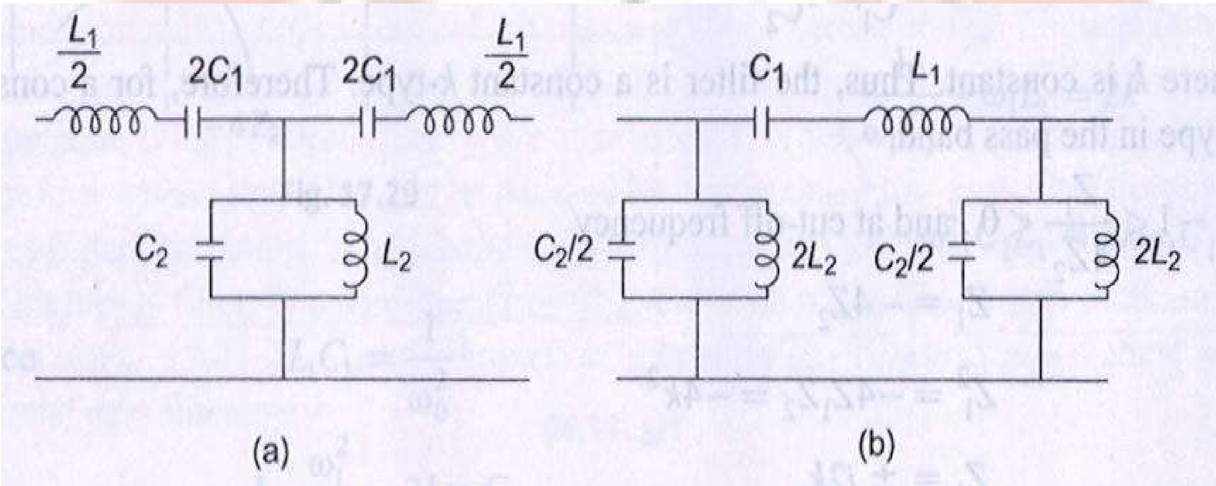


Fig.9.28

Forthisconditionofequalresonantfrequencies.

For this condition of equal resonant frequencies.

$$\omega_0 \frac{L_1}{2} = \frac{1}{2\omega_0 C_1} \quad \text{for the series arm}$$

from which, $\omega^2{}_0 L_1 C_1 = 1$

(9.34)

and $\frac{1}{\omega_0 C_2} = \omega_0 L_2$ for the shunt arm

from which, $\omega_0^2 L_2 C_2 = 1$

(9.35)

$$\omega_0^2 L_1 C_1 = 1 = \omega_0^2 L_2 C_2$$
$$L_1 C_1 = L_2 C_2$$

(9.36)

The impedance of the series arm, Z_1 is given by

$$Z_1 = \left(j\omega L_1 - \frac{j}{\omega C_1}\right) = j\left(\frac{\omega^2 L_1 C_1 - 1}{\omega C_1}\right)$$

The impedance of the shunt arm, Z_2 is given by

$$Z_2 = \frac{j\omega L_2 \frac{1}{j\omega C_2}}{j\omega L_2 + \frac{1}{j\omega C_2}} = \frac{j\omega L_2}{1 - \omega^2 L_2 C_2}$$

$$Z_1 Z_2 = j\left(\frac{\omega^2 L_1 C_1 - 1}{\omega C_1}\right)\left(\frac{j\omega L_2}{1 - \omega^2 L_2 C_2}\right)$$

$$= \frac{-L_2}{C_1}\left(\frac{\omega^2 L_1 C_1 - 1}{1 - \omega^2 L_2 C_2}\right)$$

FromEq.9.36

$$L_1 C_1 = L_2 C_2$$
$$Z_1 Z_2 = \frac{L_2}{C_1} = \frac{L_1}{C_2} = k^2$$

Where*k*isconstant.Thus, thefilterisaconstant*k*– type.Therefore,foraconstant*k*– typeinthe pass band.

$$-1 < \frac{Z_1}{4Z_2} < 0 \text{, and at cut-off frequency}$$
$$Z_1 = -4Z_2$$
$$Z_1^2 = -4Z_1 Z_2 = -4k^2$$
$$\therefore \quad Z_1 = \pm j2k$$

i.e.thevalueofZ$_1$atlowercut-offfrequencyisequaltothenegativeofthevalueofZ$_1$attheupper cut-offfrequency .

$$\therefore \quad \left(\frac{1}{j\omega_1 C_1} + j\omega_1 L_1\right) = -\left(\frac{1}{j\omega_2 C_1} + j\omega_2 L_1\right)$$

$$\text{or} \quad \left(\omega_1 L_1 - \frac{1}{\omega_1 C_1}\right) = \left(\frac{1}{\omega_2 C_1} - \omega_2 L_1\right)$$

$$(1 - \omega_1^2 L_1 C_1) = \frac{\omega_1}{\omega_2}(\omega_2^2 L_1 C_1 - 1)$$

(9.37)

FromEq.9.34,$L_1C_1=1/\omega_0^2$

HenceEq.9.37maybewrittenas

$$\left(1 - \frac{\omega_1^2}{\omega_0^2}\right) = \frac{\omega_1}{\omega_2}\left(\frac{\omega_2^2}{\omega_0^2} - 1\right)$$

$$(\omega_0^2 - \omega_1^2)\omega_2 = \omega_1(\omega_2^2 - \omega_0^2)$$

$$\omega_0^2\omega_2 - \omega_1^2\omega_2 = \omega_1\omega_2^2 - \omega_1\omega_0^2$$

$$\omega_0^2(\omega_1 + \omega_2) = \omega_1\omega_2(\omega_2 + \omega_1)$$

$$\omega_0^2 = \omega_1\omega_2$$

$$f_0 = \sqrt{f_1 f_2}$$

(9.38)

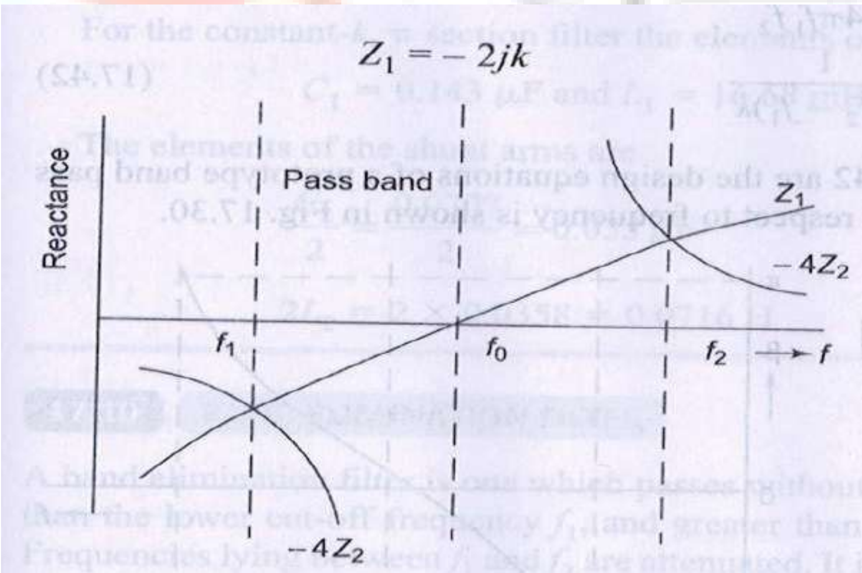


Fig.9.29

Thus,theresonantfrequencyisthegeometricmeanofthecut-offfrequencies.The variationofthereactanceswithrespecttofrequencyisshowninFig.9.29.

Ifthefilteristerminatedinaloadresistance*R*=*K*,thenatthelowercut-offfrequency.

$$\left(\frac{1}{j\omega_1 C_1} + j\omega_1 L_1\right) = -2jk$$

$$\frac{1}{\omega_1 C_1} - \omega_1 L_1 = 2k$$

$$1 - \omega^2{}_1 C_1 L_1 = 2k\omega_1 C_1$$

Since $L_1 C_1 = \dfrac{1}{\omega_0^2}$

$$1 - \frac{\omega_1^2}{\omega_0^2} = 2k\omega_1 C_1$$

or

$$1 - \left(\frac{f_1}{f_0}\right)^2 = 4\pi k f_1 C_1$$

$$1 - \frac{f_1^2}{f_1 f_2} = 4\pi k f_1 C_1 \qquad (\because f_0 = \sqrt{f_1 f_2})$$

$$f_2 - f_1 = 4\pi k f_1 f_2 C_1$$

$$C_1 = \frac{f_2 - f_1}{4\pi k f_1 f_2}$$

(9.39)

Since $L_1 C_1 = \dfrac{1}{\omega_0^2}$

$$L_1 = \frac{1}{\omega_0^2 C_1} = \frac{4\pi k f_1 f_2}{\omega_0^2 (f_2 - f_1)}$$

$$L_1 = \frac{k}{\pi (f_2 - f_1)}$$

(9.40)

To evaluate the values for the shunt arm, consider the equation

$$Z_1Z_2 = \frac{L_2}{C_1} = \frac{L_1}{C_2} = k^2$$

$$\therefore \quad L_2 = C_1k^2 = \frac{(f_2 - f_1)k}{4\pi f_1 f_2}$$

(9.41)

$$\text{and} \quad C_2 = \frac{L_1}{k^2} = \frac{1}{\pi(f_2 - f_1)k}$$

(9.42)

Equations9.39through9.42arethedesignequations ofaprototypeband passfilter.The variation of α , β with respect to frequency is shown in Fig.9.30 .

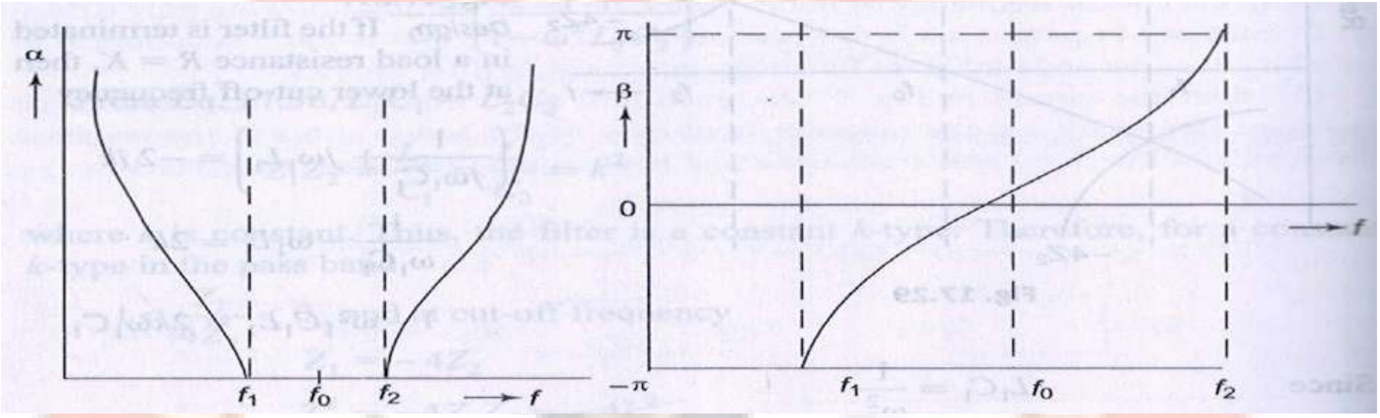


Fig.9.30

Example9.5.

Designk–typebandpassfilterhavingadesignimpedanceof500Ωandcut-off frequencies1kHzand10kHz.

Solution.

k=500Ω;f_1=1000Hz;f_2=10000Hz

FromEq.9.40,

$$L_1 = \frac{k}{\pi(f_2 - f_1)} = \frac{500}{\pi\, 9000} = \frac{55.55}{\pi}\ \text{mH} = 16.68\ \text{mH}$$

FromEq.9.39,

$$C_1 = \frac{f_2 - f_1}{4\pi k f_1 f_2} = \frac{9000}{4\times\pi\times 500\times 1000\times 10000} = 0.143\ \mu\text{F}$$

FromEq.9.41,

$$L_2 = C_1k^2 = 3.57\text{ mH}$$

FromEq.9.42,

$$C_2 = \frac{L_1}{k^2} = 0.0707\,\mu\text{F}$$

Eachofthetwoseriesarmsoftheconstant$k$,T–sectionfilterisgivenby

$$\frac{L_1}{2} = \frac{17.68}{2} = 8.84\text{ mH}$$

$$2C_1 = 2 \times 0.143 = 0.286\,\mu\text{F}$$

And the shunt arm elements of the network are given by

$$C_2 = 0.0707\,\mu\text{F and } L_2 = 3.57\text{ mH}$$

For the constant-k, π section filter the elements of the series arm are

$$C_1 = 0.143\,\mu\text{F and } L_1 = 16.68\text{ mH}$$

The elements of the shunt arms are

$$\frac{C_2}{2} = \frac{0.0707}{2} = 0.035\,\mu\text{F}$$

$$2L_2 = 2 \times 0.0358 = 0.0716\text{ H}$$

BANDELIMINATIONFILTER

Abandeliminationfilterisonewhichpasseswithoutattenuationallfrequencieslessthanthelower cut-offfrequency f_1, andgreater than the upper cut-off frequency f_2. Frequencies lying between $f_1$and $f_2$are attenuated. It is also known as band stop filter. Therefore, a band stop filter can be realized by connecting a low pass filter in parallel with a high pass section, in which the cut-off frequencyoflowpassfilterisbelowthatofa highpassfilter.Theconfigurationsof $T$and πconstant k band stop sections are shown in Fig.9.31. The band elimination filter is designed in the same manner as is the band pass filter.

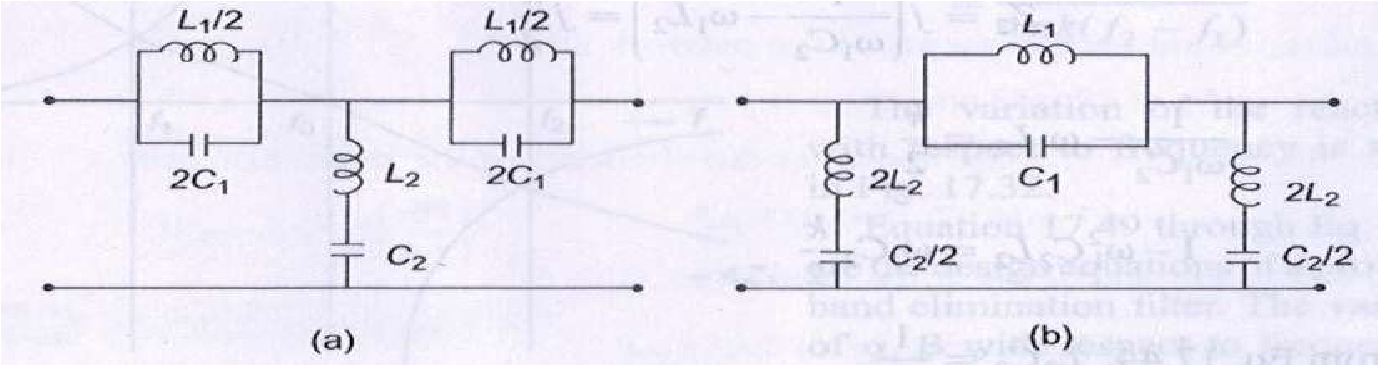


Fig.9.31

Asforthebandpass filter,theseriesandshuntarmsarechosento resonateatthesame frequencyω_0.Therefore,fromFig.9.31(a),fortheconditionofequalresonantfrequencies

$$\frac{\omega_0 L_1}{2} = \frac{1}{2\omega_0 C_1} \text{ for the series arm}$$

or $$\omega_0^2 = \frac{1}{L_1 C_1}$$

(9.43)

$$\omega_0 L_2 = \frac{1}{\omega_0 C_2} \text{ for the shunt arm}$$

$$\omega_0^2 = \frac{1}{L_2 C_2}$$

(9.44)

$$\frac{1}{L_1 C_1} = \frac{1}{L_2 C_2} = k$$

Thus $$L_1 C_1 = L_2 C_2$$

(9.45)

It can be also verified that

$$Z_1 Z_2 = \frac{L_1}{C_2} = \frac{L_2}{C_1} = k^2$$

(9.46)

and $$f_0 = \sqrt{f_1 f_2}$$

(9.47)

Atcut-offfrequencies,Z_1=- $4Z_2$

MultiplyingbothsideswithZ$_2$,weget

$$Z_1 Z_2 = -4Z_2^2 = k^2$$

$$Z_2 = \pm j\frac{k}{2}$$

(9.48)

Iftheloadisterminatedinaloadresistance,R=k,thenatlowercut-offfrequency

$$Z_2 = j\left(\frac{1}{\omega_1 C_2} - \omega_1 L_2\right) = j\frac{k}{2}$$

$$\frac{1}{\omega_1 C_2} - \omega_1 L_2 = \frac{k}{2}$$

$$1 - \omega_1^2 C_2 L_2 = \omega_1 C_2 \frac{k}{2}$$

FromEq.9.44,

$$L_2 C_2 = \frac{1}{\omega_0^2}$$

$$1 - \frac{\omega_1^2}{\omega_0^2} = \frac{k}{2}\omega_1 C_2$$

$$1 - \left(\frac{f_1}{f_0}\right)^2 = k\pi f_1 C_2$$

$$C_2 = \frac{1}{k\pi f_1}\left[1 - \left(\frac{f_1}{f_0}\right)^2\right]$$

Since $$f_0 = \sqrt{f_1 f_2}$$

$$C_2 = \frac{1}{k\pi}\left[\frac{1}{f_1} - \frac{1}{f_2}\right]$$

$$C_2 = \frac{1}{k\pi}\left[\frac{f_2 - f_1}{f_1 f_2}\right]$$

(9.49)

FromEq.9.44,

$$\omega_0^2 = \frac{1}{L_2 C_2}$$

$$L_2 = \frac{1}{\omega_0^2 C_2} = \frac{\pi k f_1 f_2}{\omega_0^2 (f_2 - f_1)}$$

Since $$f_0 = \sqrt{f_1 f_2}$$

$$L_2 = \frac{k}{4\pi(f_2 - f_1)}$$

(9.50)

AlsofromEq. 9.46,

$$k^2 = \frac{L_1}{C_2} = \frac{L_2}{C_1}$$

$$L_1 = k^2 C_2 = \frac{k}{\pi}\left(\frac{f_2 - f_1}{f_1 f_2}\right)$$

(9.51)

$$\text{and } C_1 = \frac{L_2}{k^2}$$

(9.52)

$$= \frac{1}{4\pi k(f_2 - f_1)}$$

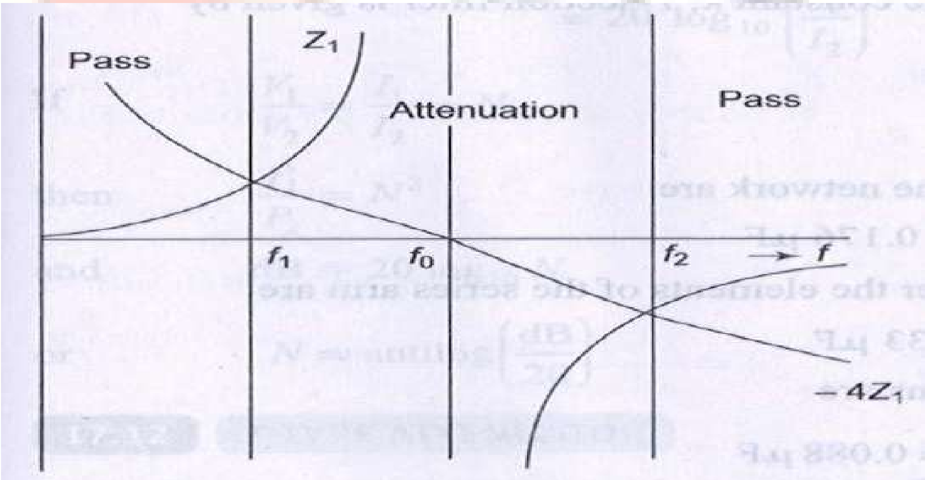


Fig.9.32

The variation of reactances with respect to frequency is shown in Fig.9.32. Equation 9.49 throughEq.9.52isthedesignequationsofaprototype bandeliminationfilter.Thevariationof α,β with respect to frequency is shown in Fig.9.33 .

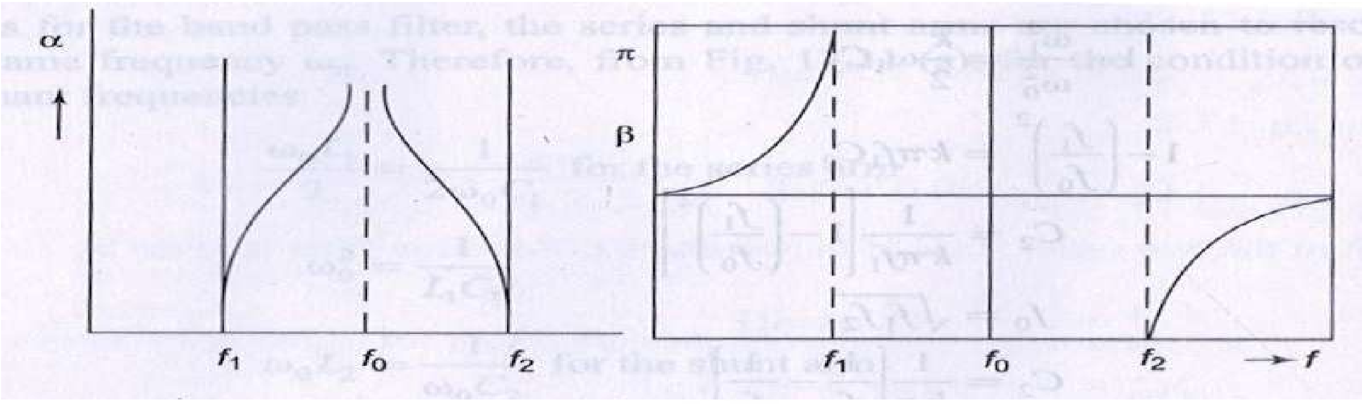


Fig.9.33

Example9.6.

Designabandeliminationfilterhavingadesignimpedanceof600Ωand cut-off frequenciesf$_1$=2kHzandf$_2$=6 kHz.

Solution.(f_2–f_1)=4kHz

MakinguseoftheEqs.9.49through9.52inSection9.10,wehave

$$L_1 = \frac{k}{\pi}\left(\frac{f_2 - f_1}{f_2 f_1}\right) = \frac{600 \times 4000}{\pi \times 2000 \times 6000} = 63 \text{ mH}$$

$$C_1 = \frac{1}{4\pi k(f_2 - f_1)} = \frac{1}{4 \times \pi \times 600(4000)} = 0.033 \ \mu\text{F}$$

$$L_2 = \frac{1}{4\pi k(f_2 - f_1)} = \frac{600}{4\pi(4000)} = 12 \text{ mH}$$

$$C_2 = \frac{1}{k\pi}\left[\frac{f_2 - f_1}{f_1 f_2}\right] = \frac{1}{600 \times \pi}\left[\frac{4000}{2000 \times 6000}\right] = 0.176 \ \mu\text{F}$$

Each of the two series arms of the constant k, T-section filter is given by

$$\frac{L_1}{2} = 31.5 \text{ mH}$$

$$2C_1 = 0.066 \ \mu\text{F}$$

And the shunt arm elements of the network are

$$L_2 = 12 \text{ mH and } C_2 = 0.176 \ \mu\text{F}$$

For the constant k, π-section filter the elements of the series arm are

$$L_1 = 63 \text{ mH}, \ C_1 = 0.033 \ \mu\text{F}$$

and the elements of the shunt arms are

$$2L_2 = 24 \text{ mH and } \frac{C_2}{2} = 0.088 \ \mu\text{F}$$